Tutorial 5: Characterization of transient distributed surface sources through infrared thermography (experimental and numerical)

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Abstract. During this workshop, we invite the participants to put into practice some of the fundamental notions seen during the courses dealing with the problem of time-space reconstruction of heat source. This tutorial focuses on the estimation of the space and time distribution of a given heat flux at the front face of a thin metallic plate starting from samples of the temperature field collected on the rear face of the material by an infrared camera. Measurements will be made during the tutorial in order to make it more realistic. The collected data will be processed and then used in a numerical code. The latter is based on a sequential estimation method: the state representation "pseudo-inversion" of the 3D parabolic model of heat conduction in the material. Of course, the parameterized heat flux distribution is discrete in both space (2D) and time. A good space and time resolution requires a high number of unknown values. Since the estimation problem is linear, it is not necessary to use an iterative method with a stopping criterion acting as a regularization hyperparameter. In the case considered, the high number of unknowns to be estimated simultaneously as well as the measurement noise makes the ill-posed character of this multi-dimensional inverse problem quite high: regularisation tools are highly recommended in such situations. Regularization is implemented through optimization under constraints: it allows shading off the inherent instabilities of the heat flux solution. We suggest testing two regularizing techniques: (i) the stabilization by the function specification method proposed by Beck and (ii) the regularization by penalization developed by Tikhonov, with three orders: 0, 1 and 2.

1. Introduction

The heat flux estimation by inverse method has been of great interest for many years now. It is often encountered when thermal systems have to be characterized and measures are impossible to take. For instance the location and intensity of heat sources in electronic components must be well known to predict the electrical behaviour of the devices. In the same way inverse method would be used to evaluate real contact areas in solid-solid contacts.

Estimation of time and space dependent heat flux without a priori information on the solution is still a difficult task. Indeed, the time and space discretization of the sought values can lead to a very high number of unknown values and the inverse problem is then very ill-posed.

During this workshop a method to evaluate the time and space dependant heat flux is shown. It is proposed to evaluate it at the front face of a thin metallic plate starting from samples of temperature field collected on the rear face of the material by an infrared camera. For this purpose a sequential method regularized by a procedure that involves two well-known techniques: the stabilization by the function specification method proposed by Beck and the regularization by penalization developed by Tikhonov, with three orders: 0, 1 and 2.

Inversion of experimental data, made during this tutorial, confirms the ability of the algorithm to reconstitute the time and space dependant heat flux with accuracy.

2. The experimental test bench

A square shaped thin steel plate (k, ρ, c_p) of dimensions 48x48x4[mm] is considered. Two Vishay heaters, connected to a power supplier, are fixed on the front face (see figure 2). The heated face is placed on a thermal insulator. A Flir A40 infrared camera collects the temperature field of the plate rear face that was painted in black. The camera was calibrated earlier using thermocouples.

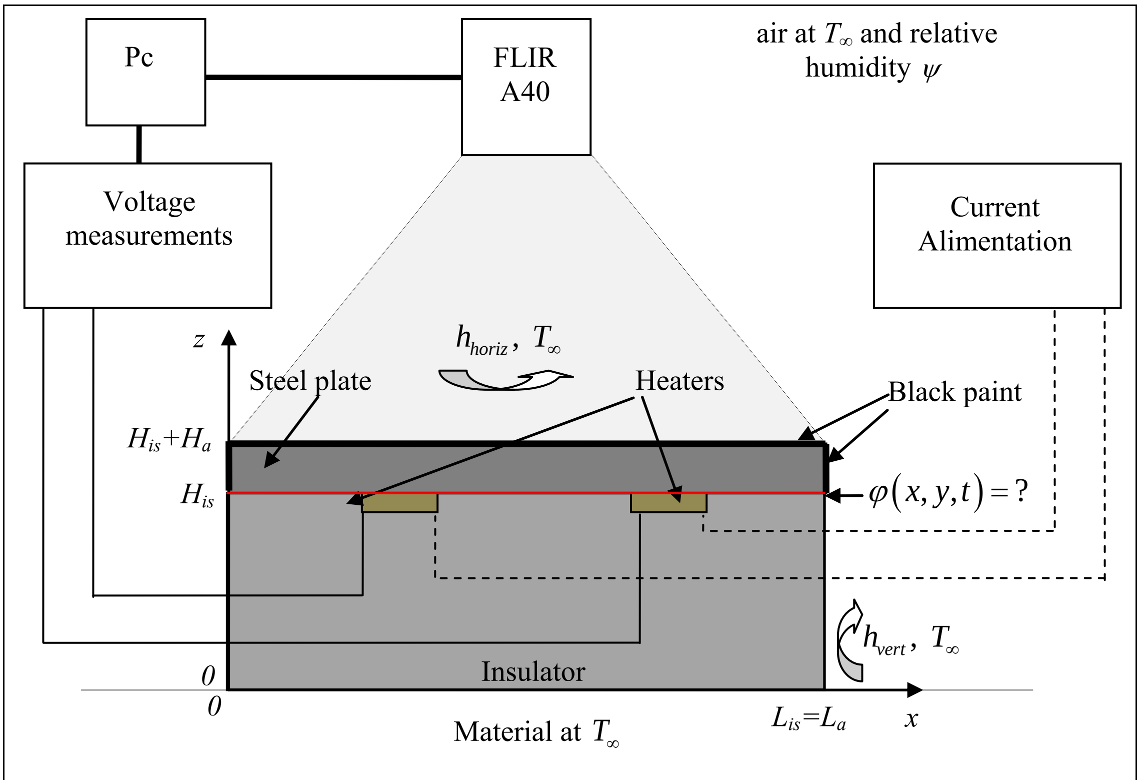

Figure 1. Sketch of experimental test bench

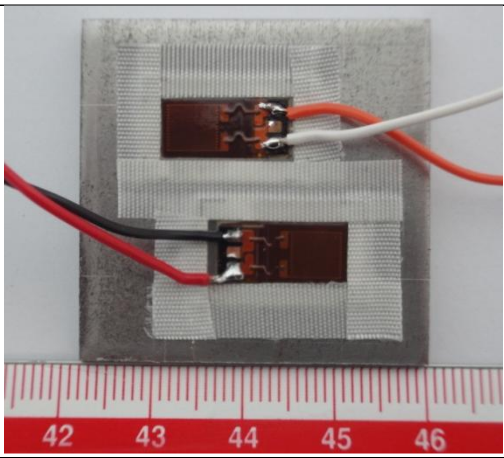

Figure 2. Pictures of heaters

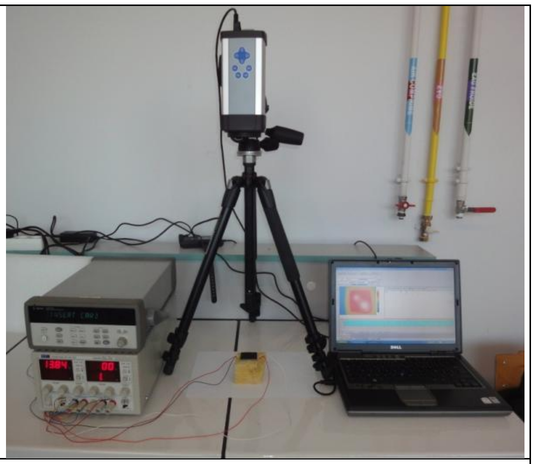

Figure 3. Pictures of experimental test bench elements

3. Measures treatment

The collected data are firstly converted into Matlab format and then gathered in a matrix. After that the following steps are performed:

1) Plate cutting-out in all pictures
2) First picture subtraction to all others: in the first picture nothing happened so that the plate is at a uniform temperature.
3) First picture mean temperature addition to all others.
4) Data filtering: the inversion algorithm is based on the finite volume discretization of heat equation. As the spatial resolution of the camera is more refined than the chosen mesh as well as the frequency acquisition vs calculation time step, meaning data on the mesh and the time step as a filtering effect and it contribute to the regularization.
5) Use of calibrating function.

Once these steps are done, data are injected into the inversion algorithm.

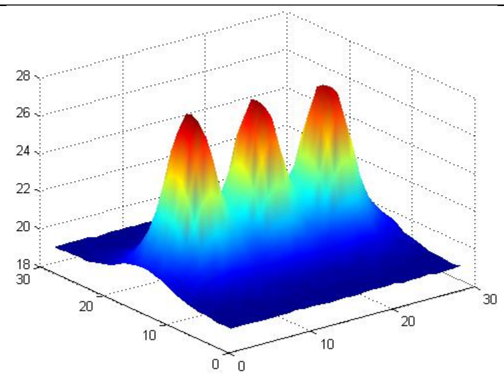

Figure 4. Measured temperature in *°C* (rear face; case of three heaters)

4. Algorithm

The inversion algorithm is based on a sequential estimation method: the state representation "pseudo-inversion" of the 3D parabolic model of heat conduction in the material. This representation comes from the finite volume discretization and the time scheme is full implicit. This stage is not detailed here because it is not the topic of this workshop. It allows to put the heat conduction problem into the state matrix equation (1):

$$\forall n = 0,1 ... Nn, \qquad \underline{\underline{\mathbf{A}}}\underline{T}_{n+1} + \underline{\underline{\mathbf{A}}}'\underline{T}_n + \underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1} + \underline{\underline{\mathbf{B}}}_{\mathbf{nc}}\underline{\varphi}_{n+1} = 0 \tag{1}$$

The initial field $\underline{T}_1$ is the result of a steady state regime which is found by introducing the arbitrary fields $\underline{T}_0$ and $\underline{U}_{c,0}$ such as:

$$\underline{T}_0 = \underline{T}_1,\ \underline{U}_{c,0} = \underline{U}_{c,1} \text{ and } \underline{\varphi}_0 = \underline{\varphi}_1 \tag{2}$$

- $\underline{T}_{n+1}$ ($\dim(Nm)$) is the temperature vector at time $n\Delta t$ (it contains the values of the temperature in the mesh cells)
- Δt is the time step i.e. the elapsed time between times n and $n+1$.
- $\underline{U}_{c,n+1}$ ($\dim(Npc)$) is the known solicitation vector at time $(n+1)\Delta t$.
- $\underline{\varphi}_{n+1}$ ($\dim(Npnc)$) is the unknown solicitation vector at time $(n+1)\Delta t$.
- $\underline{\underline{\mathbf{A}}}$ and $\underline{\underline{\mathbf{A}}}'$ ($\dim(Nm, Nm)$) are evolution matrix.
- $\underline{\underline{\mathbf{B}}}_c$ ($\dim(Nm, Npc)$) is the command matrix of known solicitations.
- $\underline{\underline{\mathbf{B}}}_{nc}$ ($\dim(Nm, Npnc)$) is the command matrix of unknown solicitations.
- Nm is the mesh cell number.
- Npc is the known solicitation number.
- $Npnc$ is the unknown solicitation number.

Let's define $\underline{\underline{C}}$ ($\dim(Nq, Nm)$) the observation matrix which allows to select the Nq temperatures of vector $\underline{T}_n$ to compare to the measures $\underline{Y}_n^*$:

$$\underline{Y}_n = \underline{\underline{C}}\underline{T}_n \tag{3}$$

Using relations (3) and (1) we can write:

$$\forall n = 0 ... Nn, \qquad \underline{Y}_{n+1} = \underline{\underline{C}}\underline{T}_{n+1} = \underline{\underline{C}}\underline{\underline{\mathbf{A}}}\underline{T}_n + \underline{\underline{C}}\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1} + \underline{\underline{C}}\underline{\underline{\mathbf{B}}}_{\mathbf{nc}}\underline{\varphi}_{n+1} \tag{4}$$

$$\text{with : } \qquad \underline{\underline{\mathbf{A}}} = -\underline{\underline{\mathbf{A}}}^{-1}\underline{\underline{\mathbf{A}}}',\ \underline{\underline{\mathbf{B}}}_{\mathbf{c}} = -\underline{\underline{\mathbf{A}}}^{-1}\underline{\underline{\mathbf{B}}}_{\mathbf{c}} \text{ and } \underline{\underline{\mathbf{B}}}_{\mathbf{nc}} = -\underline{\underline{\mathbf{A}}}^{-1}\underline{\underline{\mathbf{B}}}_{\mathbf{nc}} \tag{5}$$

It is chosen to find the heat flux vector by minimizing the function *S* (6) that calculate the root means square between the measured and calculated temperatures at the rear of the material.

$$S = \sum_{n=1}^{Nn+1} S_n = \sum_{n=1}^{Nn+1} \left\| \underline{Y}_n - \underline{Y}_n^* \right\|^2 \tag{6}$$

It can be shown that the minimization of *S* leads to (7) for the unknown vector:

$$\forall n = 0...Nn, \qquad \underline{\hat{\varphi}}_{n+1} = \left[\underline{\underline{D}}^T \underline{\underline{D}}\right]^{-1} \underline{\underline{D}}^T \left[\underline{Y}^*_{n+1} - \underline{b}_{n+1}\right] \tag{7}$$

with :
$$\underline{\underline{D}} = \underline{\underline{C}}\,\underline{\underline{\mathbf{B}}}_{\mathbf{nc}} \text{ and } \underline{b}_{n+1} = \underline{\underline{C}}\,\underline{\underline{\mathbf{A}}}\,\underline{T}_n + \underline{\underline{C}}\,\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1} \tag{8}$$

- In this application $Nq = Npnc$.
- The reason why we call the method "peudo-inversion of state representation" is that the matrix $\left[\underline{\underline{D}}^T \underline{\underline{D}}\right]^{-1} \underline{\underline{D}}^T$ is often called "pseudo-inverse" of $\underline{\underline{D}}$.
- The illness posed character is shown by a bad conditioning of the matrix $\underline{\underline{D}}^T \underline{\underline{D}}$.

5. Regularisation

5.1. Futur times steps (Beck)

The future time step technique is one of the most used for the temporal stabilization of the transient inverse heat conduction problems (IHCP). This method of function specification was introduced by Beck [1] and developed by numerous others. It is essentially used during the sequential estimation of temperature and density of heat flow. It consists in overdetermining the system to resolve by information addition. More exactly, it uses measured information at moments later than the time when the unknowns are estimated. It assumes a temporary hypothesis on the evolution of the sought values. The interval of time on which the hypothesis is valid corresponds among step of time future *Nf*. The simplest hypothesis is to consider constant parameters during future times. The function to be minimized becomes:

$$\forall n = 0...\left[Nn + 1 - Nf\right], \qquad S_n = \sum_{f=0}^{Nf} \left\| Y_{n+f} - Y^*_{n+f} \right\|^2 \tag{9}$$

To minimize the function (9) it is supposed that:

$$\forall f = 0...Nf, \qquad \underline{\varphi}_{nc,n+f} = \underline{\varphi}_{nc,n} \quad \text{for} \quad 1 \le f \le Nf \tag{10}$$

It can be shown that the system (4) is then replaced by the following globalized system:

$\forall n = 1...[Nn - Nf]$, $$\underline{\mathbf{Y}}_{n+1} = \underline{\underline{\mathbf{D}}}\underline{\varphi}_{n+1} + \underline{\mathbf{b}}_{n+1} \tag{11}$$

where: $$\underline{\mathbf{Y}}_{n+1} = \begin{bmatrix} \underline{Y}_{n+1} \\ \underline{Y}_{n+1+1} \\ \vdots \\ \underline{Y}_{n+1+f} \\ \vdots \\ \underline{Y}_{n+1+Nf} \end{bmatrix} \quad \underline{\underline{\mathbf{D}}} = \begin{bmatrix} \underline{\underline{\mathbf{D}}}_{n+1} \\ \underline{\underline{\mathbf{D}}}_{n+1+1} \\ \vdots \\ \underline{\underline{\mathbf{D}}}_{n+1+f} \\ \vdots \\ \underline{\underline{\mathbf{D}}}_{n+1+Nf} \end{bmatrix} \quad \underline{\mathbf{b}}_{n+1} = \begin{bmatrix} \underline{\mathbf{b}}_{n+1} \\ \underline{\mathbf{b}}_{n+1+1} \\ \vdots \\ \underline{\mathbf{b}}_{n+1+f} \\ \vdots \\ \underline{\mathbf{b}}_{n+1+Nf} \end{bmatrix} \tag{12}$$

with: $$\underline{\underline{\mathbf{D}}}_{n+1+f} = \underline{\underline{C}} \sum_{j=0}^{j=f} \underline{\underline{\mathbf{A}}}^{j} \underline{\underline{\mathbf{B}}}_{\mathbf{nc}} \tag{13}$$

and $$\underline{\mathbf{b}}_{n+1+f} = \underline{\underline{C}}\,\underline{\underline{\mathbf{A}}}^{f+1} \underline{T}_n + \underline{\underline{C}} \sum_{j=0}^{j=f} \underline{\underline{\mathbf{A}}}^{j} \underline{\underline{\mathbf{B}}}_{\mathbf{c}} \underline{U}_{c,n+1+f-j} \tag{14}$$

Even if $Nq = Npnc$, the system (11) is overdetermined as soon as $Nf \neq 0$. It indeed possesses $Nq \times (Nf + 1)$ equations with $Npnc$ unknowns. Its resolution in the least mean square sense leads to (15):

$\forall n = 1...[Nn - Nf]$, $$\hat{\underline{\varphi}}_{n+1} = \left[\underline{\underline{\mathbf{D}}}^T \underline{\underline{\mathbf{D}}}\right]^{-1} \underline{\underline{\mathbf{D}}}^T \left[\underline{\mathbf{Y}}^*_{n+1} - \underline{\mathbf{b}}_{n+1}\right] \tag{15}$$

- If $Nf = 0$, the relation (7) is obviously found.
- The solution at the initial time is a particular case: as it is the result of a steady state regime there is no future time step.

5.2. Tikhonov penalization

In heat transfer the regularization by Tikhonov penalization [2] is one of the most wide-spreaded. It consists in adding an a priori information about the solution by tending to impose predetermined values or laws to the solution behavior. It presents the advantage that the resolution techniques of the inverse problem stay the same than if the problem was not regularized. This regularization consists in minimizing the following function:

$$S = \sum_{n=1}^{Nn+1} \left[\sum_{f=0}^{Nf} \left\| Y_{n+f} - Y^*_{n+f} \right\|^2 + \mu_{T,n} \left\| \underline{\underline{R}} \underline{\varphi}_n \right\|^2 \right] \tag{16}$$

In that case the minimization of S in the least mean square sense leads to:

$\forall n = 1...[Nn - Nf]$, $$\hat{\underline{\varphi}}_{n+1} = \left[\underline{\underline{\mathbf{D}}}^T \underline{\underline{\mathbf{D}}} + \mu_{T,n+1} \underline{\underline{R}}^T \underline{\underline{R}}\right]^{-1} \underline{\underline{\mathbf{D}}}^T \left[\underline{\mathbf{Y}}^*_{n+1} - \underline{\mathbf{b}}_{n+1}\right] \tag{17}$$

And effectively the resolution of the inverse problem is only little modified.

- $\mu_{T,n}$ is a parameter that allows to adjust the regularization. If it is chosen identical for all times, we note it μ_T .
- $\underline{\underline{R}}$ is a matrix that depends on the type of chosen regularization. It contains information a priori on the solution more exactly on the "links" which there is, or which we suppose that there is, between the components of $\underline{\varphi_n}$. The simplest regularization consists in choosing a regularization of order 0 with $\underline{\underline{R}} = \underline{\underline{I}}$. When the regularization parameter $\mu_{T,n}$ is low, the regularization has not enough influence. On the other hand, when it increases, the regularization tends to impose values on the components of the solution vector. For example with a regularization of order 0, null values tend to be imposed to the unknowns. The regularization has then an effect of smoothing the IHCP solution. The key point of this method is thus the choice of the parameter of regularization because if it is badly chosen, it can distort the problem and the solution will be biased. The optimal parameter $\mu_{T,n}$ will minimize the error of prediction of the unknowns considering the noise of measures. Various criteria of choice are proposed. Generally, the choice of $\mu_{T,n}$ is based on the discrepancy principle: we look for $\mu_{T,n}$ such as the RMS distance S between the measured temperatures and those worked out again with the direct problem from the estimated values are of the same order size as the variance of the measurement noise. Here, the search for the parameter is realized in an iterative way.

As talked about in the previous paragraph, the Tikhonov regularization of order 0 tends to impose null values on solution vectors. This regularization tends to smooth the solution. To limit this smoothing, we can turn to a regularization of order 1. This regularization consists in minimizing the function (18):

$$S = \sum_{n=1}^{Nn+1} \left[\left\| \underline{Y}_n - \underline{Y}_n^* \right\|^2 + \mu_{T,n} \left\| \nabla_{x,y} \underline{\varphi_n} \right\|^2 \right] \tag{18}$$

where
$$\mu_{T,n} \left\| \nabla \underline{\varphi_n} \right\|_2^2 = \mu_{T,n} \left[\underline{\varphi_n} \right]^T \underline{\underline{R}} \left[\underline{\varphi_n} \right] \tag{19}$$

with
$$\underline{\underline{R}} = \left[\underline{\underline{R}}_x \right]^T \underline{\underline{R}}_x + \left[\underline{\underline{R}}_y \right]^T \underline{\underline{R}}_y \tag{20}$$

- $\underline{\underline{R}}_x$ is the discrete operator that derive $\underline{\varphi_n}$ in x direction.
- $\underline{\underline{R}}_y$ is the discrete operator that derive $\underline{\varphi_n}$ in y direction.

This time we tend to uniformise the solution in space. The regularization of order 2 consists in taking operators who derive twice $\underline{\varphi_n}$ in the space directions. It thus tends to impose a linear relation between the components of $\underline{\varphi_n}$.

6. Measures inversion

These algorithms are implemented using Matlab software. The data treated earlier are used in these algorithms, first without regularization and then with regularization.

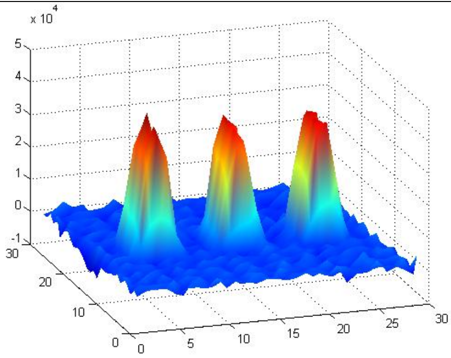

Figure 5. Estimated heat flux in $W.m^{-2}$ (front face; case of three heaters)

7. Conclusion

Measurements are done during the workshop. Data are treated and then used in an algorithm to calculate the distributions in space and time of a given heat flux at the front face of a thin metallic plate starting from samples of the temperature field collected on the rear face of the sample by an infrared camera. Two regularizing techniques are suggested to be used: (i) the stabilization by the function specification method proposed by Beck and (ii) the regularization by penalization developed by Tikhonov with three orders: 0, 1 and 2. The results show that the algorithm allows estimating the unknown values with accuracy.

8. References

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