METTI5 Tutorial T3 on:

"Thermal characterization of materials through the hot wire/hot plate techniques"

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Duration:

1h00

Type:

Experimental

Nomenclature:

r, x - spatial coordinates (radius, distance) [m]
t - time [s]
a - thermal diffusivity [m^2 s^{-1}]
λ - thermal conductivity [W m^{-1} K^{-1}]
C_p - specific heat [J kg^{-1} K^{-1}]
ρ - mass density [kg m^{-3}]
T - temperature [K]
ψ - heat flux density [W m^{-2}]

1. Hot wire and hot plate: quasi stationary regime methods for thermal characterization

These methods are based on quasi stationary heat transfer in the sample. They are common in the industrial practice due to their simplicity and low cost. The identification of the thermal parameter is generally done at the long times, nearby the stationary regime. The different geometry of the sample and that of the sensor leads to different thermal parameters.

This kind of metrology uses the sensor composed of heating resistance and the thermocouple for temperature measurement at the sensor location.

2. Hot wire method

The hot wire method is based on simple principle and demands basic equipment. For these characteristics it is used widely in the industrial practices. This method was introduced at the beginning of the 30'ies of the precedent century by B. Stahlane et S. Pyk [1] for measurements of granular media. Nowadays it is used for solids and liquids also.

The experimental device is composed of a metallic wire immersed in liquid or introduced between two pieces of solid sample, depending of the studied media (Figure 1). Using an electric power supply, the wire is supplied with a constant electric potential. Due to the Joule effect, the wire is heating and the temperature evolution is measured at its centre by a thermocouple. The second way of detection of the temperature is using the variation of electrical resistance of the wire.

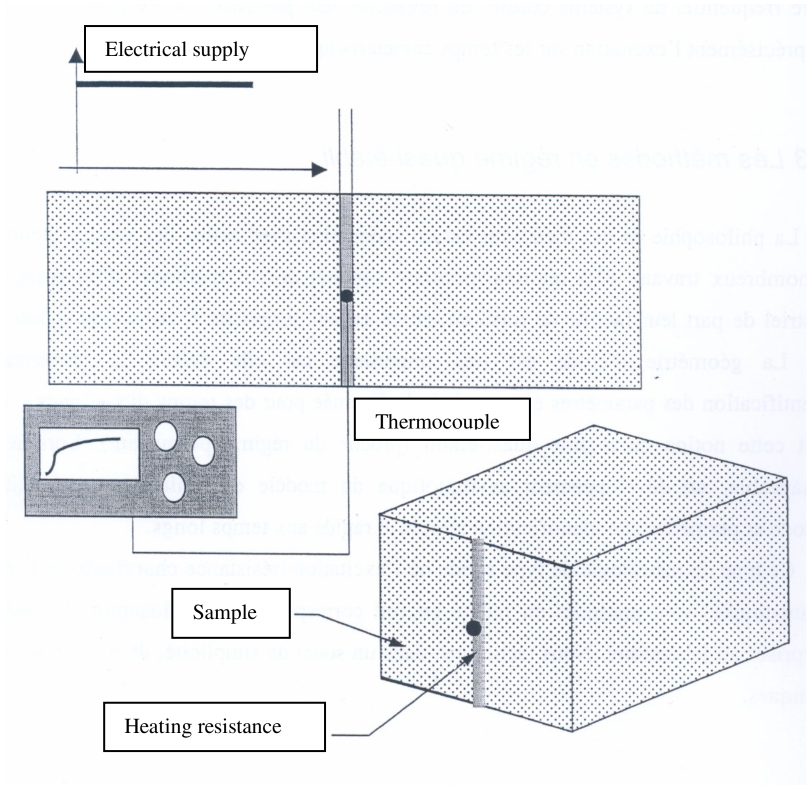

Figure 1: The hot wire method principle.

The modeling of thermal behavior of the presented system uses some assumptions:

- The wire is infinitely long
- The heat flux dissipated by the wire is radial and only conductive
- The samples are considered like infinite during the measurement
- The thermal properties of the material are constant during the experiment

In such a case the equation of heat conduction in the sample in cylindrical coordinates is:

$$\frac{\partial^2 T(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,t)}{\partial r}=\frac{1}{a}\frac{\partial T(r,t)}{\partial t} \tag{1}$$

Another assumption is that the wire is infinitely thin. However in the practice it is difficult to achieve. The boundary condition in this approximation is thus:

$$-\lambda \left.\frac{\partial T(r,t)}{\partial r}\right|_{r=0}=\psi \tag{2}$$

where ψ is the heat flux density generated by the hot wire.

The samples are considered as infinite during the experiment, the boundary condition is thus:

$$T(r\rightarrow\infty,t)=T_\infty \tag{3}$$

The initial condition is:

$$T(r,t=0)=T_\infty \tag{4}$$

The solution of this system of equation leads to the temperature in the sample:

$$T(r,t) = \frac{\psi}{4\pi\lambda} \mathrm{Ei}\left(\frac{r^2}{4at}\right) + T_\infty \tag{5}$$

with Ei exponential integral function.

When $r^2/4at$ is small, the development of this function at the proximity of 0 leads at $r = r_0$ to:

$$T(r,t) = \frac{\psi}{4\pi\lambda}\left(\ln\left(\frac{r^2}{4at}\right) + \frac{r_0^2}{4at} + \cdots\right) + T_\infty \tag{6}$$

At the long times, the relation (6) can be simplified to give the temperature of the hot wire in function of logarithm of the time. Knowing the heat flux generated by the wire, the thermal conductivity of the sample can be identified by linear regression:

$$T(r,t) - T_\infty = \frac{\psi}{4\pi\lambda} \ln t + \ln\left(\frac{4a}{r_0^2 C}\right) \tag{7}$$

This very simple method was improved by taking into account the effects of the sensor. The heat capacity and the thermal resistance between the sensor and the sample can be modelled in order to describe more precisely the thermal system. These points will be discussed during the tutorial.

2. The hot plate method

The principle of this technique is similar to the hot wire technique. The geometry of the sensor is different, in this case plane and not cylindrical. This technique was proposed by P. Vermote in 1937 [2] (Figure 2). This method leads to the thermal conductivity and the thermal diffusivity using the asymptotic solutions for the sensor temperature at the small and at the log times. The two expressions come from the solution of one dimensional heat conduction problem in the sample.

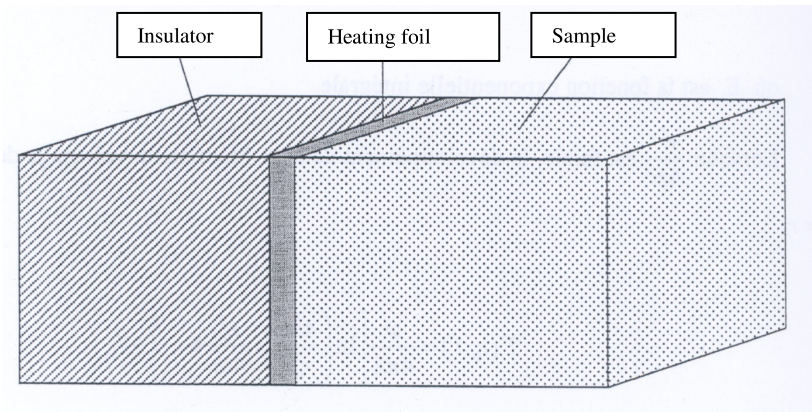

Figure 2: The hot plate measurement principle.

Some developments of this method led to simpler experimental setups permitting avoiding of heat losses or imposing a constant temperature and not heat flux. Among the most known are the symmetrical setups for measurements of insulators (Figure 3).

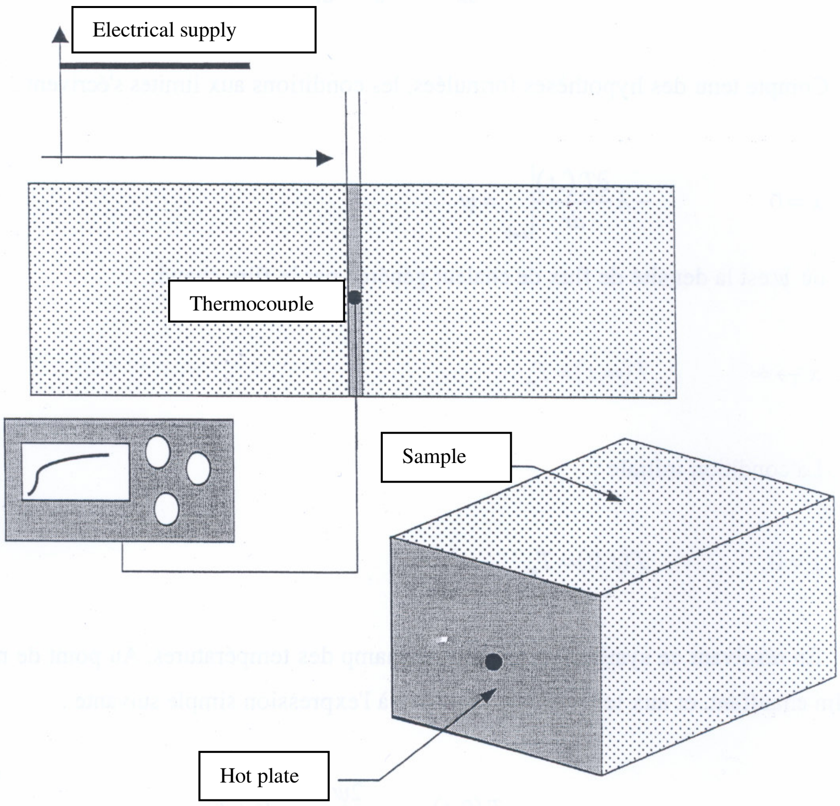

Figure 3: Symmetrical setup using the hot plate measurement.

As for the hot wire, the modeling of the thermal behavior of the sample in the hot plate measurement requires some assumptions:

- The sensor has the same dimensions as the sample
- The heat flux is only conductive and in one direction
- The two samples are symmetrical and have infinite thickness during the measurement
- The thermal properties of the sample are constant during the measurement

The equation in Cartesian coordinates describing the heat conduction in the sample is:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{a}\frac{\partial T(x,t)}{\partial t} \tag{8}$$

Considering the above assumptions the boundary conditions are:

$$-\lambda \left.\frac{\partial T(r,t)}{\partial x}\right|_{x=0} = \psi \tag{9}$$

with ψ the heat flux density dissipated by the hot plate, and :

$$T(x \to \infty, t) = T_\infty \tag{10}$$

The initial condition is:

$$T(r, t=0) = T_\infty \tag{11}$$

The solution of the above system of equations leads to the temperature field. The temperature measured on the sensor at the long times is then:

$$T(0,t) = \frac{2\psi}{\sqrt{\pi}\sqrt{\lambda \rho C_p}} \sqrt{t} + T_\infty \qquad (12)$$

Knowing the heat flux dissipated by the sensor and the temperature on its surface a simple linear regression permit obtaining the thermal effusivity of the studied sample.

This method has several drawbacks. The heat losses on the lateral faces of the sample are not taken into account. The influence of the sensor may not be negligible also. Some modeling can improve the performance of the method [3].

3. References

[1] B. STALHANE and S. PYK, Teknish Tidskrift, Vol 61, pp 389-393, 1931.
[2] P. VERMOTTE, C.R. Acd., Paris, Vol 204, P 563,1937
[3] S.E. GUSTAFSSON, Transient plane source technique for thermal conductivity and thermal diffusivity measurements of solids materials, Rev. Sci. Instrum., Vol 62, pp797-804, 1991