Tutorial 11

Inverse Heat Conduction Problem using thermocouples deconvolution and infrared measurements: application to heat flux estimation in a Tokamak

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Abstract. This tutorial is especially designed to the beginners in inverse techniques in heat conduction. Internal components of magnetic confinement fusion machines are subjected to significant heat fluxes. In order to estimate the input heat flux on these plasma-facing components, some temperature measurements techniques are used: IR scanner, embedded thermocouples. Through this experimental example; we propose to detail a heat flux estimation procedure associating deconvolution and regularization method (Tikhonov). After a brief presentation of the experimental context and the localisation of the inverse problem, the inversion procedure used and validity domain of the method will be presented. Finally, experimental cases will be treated and presented.

1. Introduction

Internal components of magnetic confinement fusion machines are subjected to significant heat fluxes. As an example, in the Joint European Torus (JET), several MW are coupled to plasma facing components for about 10 seconds [1]. A large part of this power is directed towards inertially cooled carbon tiles. In JET experiments, for better understanding and control of the heat transfer from the plasma to the surrounding wall, it is very important to measure the surface temperature of the target tiles and to estimate the imposed heat flux. That will be even more important for the protection of the internal components of the International Thermonuclear Experimental Reactor (ITER). In the JET tokamak, an infrared system and several embedded thermocouples are used to measure respectively the surface and bulk temperatures of the carbon composite tiles [2]. In the JET experiment and in most of the Tokamaks using carbon plasma facing components (PFC), the eroded carbon is circulating in the plasma and is redeposited elsewhere. During the plasma operation, this leads at some locations to the formation of thin or thick carbon layers usually poorly attached to the PFC. These surface layers complicate the calculation of the heat flux from IR surface temperature measurements. The advantage of using the temperature data of embedded thermocouples is that they are not sensitive to the surface layer. Their disadvantage is that they have poor spatial resolution. We overcome this limitation with the help of another diagnostic giving the spatial shape of the heat flux. In the case of thermocouples, the temperature measurement is distant from the tile surface then, heat flux estimation becomes an inverse problem [3]. So, we need to use more complicated numerical methods to solve the heat conduction equation.

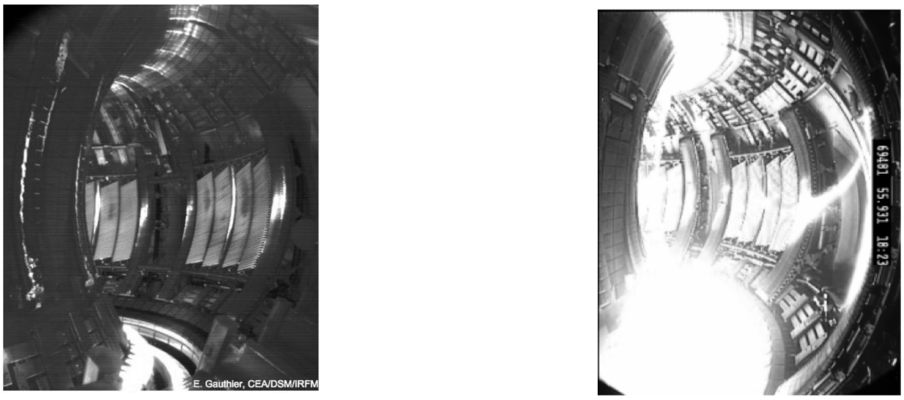

Figure 1. IR and visible view of the JET Tokamak.

The first part of the tutorial is devoted to the experimental set-up and the problematic of the heat flux estimation in a Tokamak. Then, we describe the using methodology to estimate a heat flux with an embedded measurement. We apply the method in the case of a 1D material submitted to a known heat flux. Finally, we come back on our problematic in the Tokamak JET to estimate a heat flux depending on time and space with only one thermocouple measurement and information on the spatial shape.

2. Experimental Set-Up, Problematic

2.1. Experimental Set-Up

In JET, most of the plasma facing components are tile-like structures (see Figure 2,3 and 4) made of carbon fiber composite (CFC). The tile studied in this paper and the thermocouple location are presented in Figure 3 and 4, x represents the poloïdal direction, y the toroïdal direction and z the depth in the tile. The tile is located in the divertor, which is the bottom of the machine. Tile surface temperatures are measured by an infrared camera sensitive in the 3-5µm wavelength range. The IR camera measures the radiative flux emitted by the tile surface, the temperature is then deduced with the black-body calibration of the IR camera and the knowledge of carbon emissivity (near to 0.83). An example of infrared image is presented in Figure 2b. The space resolution is about 8-10 mm per pixel, time resolution for the discharge analysed in this tutorial is 15.9 ms between two frames. Two internal temperatures are measured by two type K thermocouples (Figure 3). The thermocouple locations are (x = 25 mm, z = 10 mm) and (x = 125 mm, z = 10 mm) in the tile [2], which means they are located at 10 mm of the tile surface.

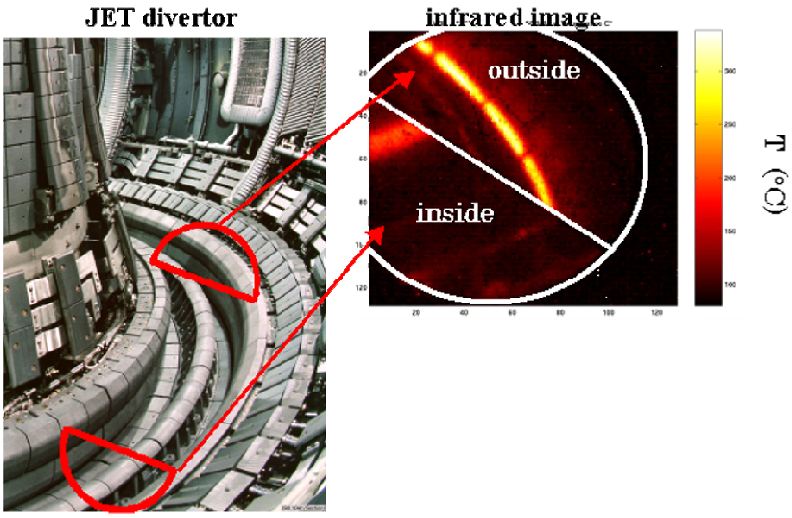

Figure 2. Vision of the JET Chamber

a) Visible view of the JET chamber **b)** Infrared image of the divertor tiles

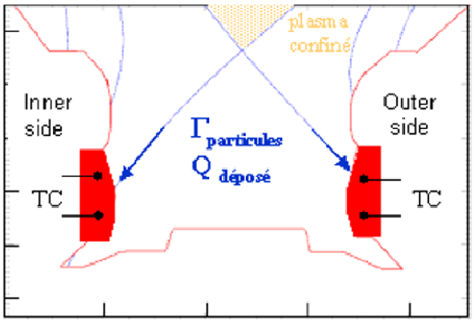

Figure 3. Poloïdal cut of JET divertor showing the magnetic field lines and carbon tiles

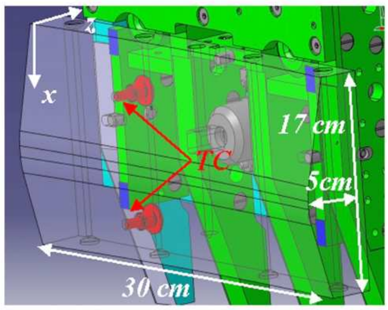

Figure 4. Dimensions of the tile n°7 of the MKIIGB divertor

2.2. Problematic

The problem consists in estimating the heat flux deposited on several tiles of the divertor. With an IR diagnostic measuring the surface temperature, the heat flux computation is not an inverse problem since the measurements can be applied as Dirichlet's conditions in a classical finite element method computation or another computation technique. Unfortunately, a very thin carbon layer can be deposited on the tile with unknown thermal properties and the using of the IR measurements as Dirichlet's condition becomes impossible. So, we have to find another way to estimate the heat flux. It's possible to use the embedded thermocouples. In this case, the temperature measurement is now embedded in the bulk and the heat flux estimation becomes an inverse problem. Moreover, the spatial resolution of the thermocouple is insufficient; we use another diagnostic to obtain the spatial shape of the heat flux.

3. Description of the method, application to a 1D Inverse Heat Conduction Problem

3.1. Presentation of the direct problem

Considering a 1D material with constant thermal properties (λ = 240W/mK, ρ=1800kg/m^3, Cp=780J/kg.K, e=0.04m) submitted to a heat flux step of 1W/m^2 between 5 and 10 seconds, we can compute the temperature for several depths in the material (z=0, 1, 2, 3, 4 cm) with a direct calculation (FEM, thermal quadrupoles, analytical solution).
Rmq: during the presentation, we will work with a semi-infinite modelling in order to have an analytical solution. The principle of the deconvolution procedure is the same.

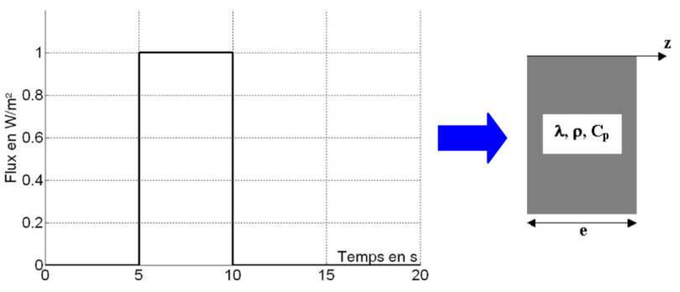

Figure 5. Heat flux applied to the bulk of e thickness

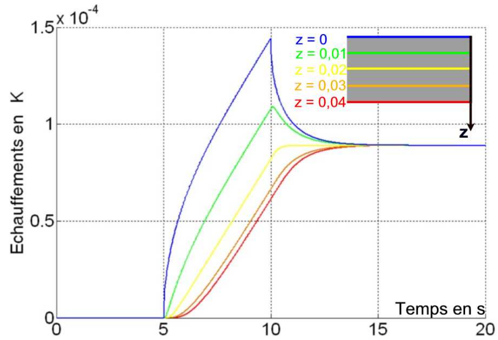

Figure 6. Bulk heating in K for different thicknesses: z=0, z=0.01m, z=0.02m, z=0.03m, z=0.04m.

One can note that all the heating have the same values after 15s. It corresponds to the temperature of the adiabatic stability. In our case, this value is equal to:

$$\Delta T_{stab} = T_{stab} - T_0 = \frac{E}{\rho.Cp.e} = \frac{5\times1}{1800\times780\times0.04} = 8,903.10^{-5}K \qquad (1)$$

E is the energy injected in the bulk in J/m².

A noise is added to the numerical signal in order to obtain more realistic data to inverse, the new signal can be written:

$$Y = Ynum + \varepsilon \qquad (2)$$

Y is the new signal.
Ynum is the numerical signal.
ε is a centred zero mean, Gaussian noise with a standard deviation of 10% of the maximal heating.

3.2. Deconvolution procedure description

The carbon tile is modelled by a linear system subjected to a prescribed heat flux Q(z=0,t) having for effect the temperature T(z,t). The linear system theory allows to express the temperature T(z,t) by the convolution of Q(z=0,t) with the pulse response h(z,t) of the system, (i.e. the tile temperature response after a delta function (Dirac function) of power applied to the surface). We assume the temperature homogeneity of the tile at t = 0.

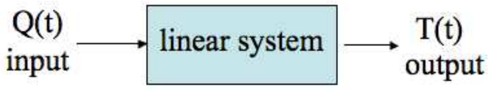

Figure 7. Linear System.

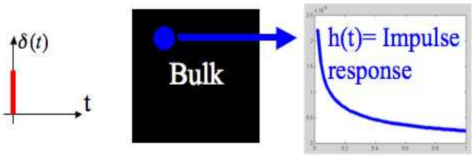

Figure 8. Impulse response of the bulk.

For the temperature T at the time t, the depth z:

$$T(z,t) = T(z,t=0) + Q(z=0,t) \otimes h(z,t) = T(z,t=0) + \int_{t_0}^{t} Q(z=0,\tau)h(z,t-\tau)d\tau \tag{3}$$

The pulse response h(z,t) of the system is the first time derivative of its step response u(z,t). So, we approximate (3) by finite differences which leads to the expression of the temperature at each time step F in matrix form: where X is a triangular lower square matrix (of order F) assembled with the components $\Delta u(z,F) = u(z,F) - u(z,F-1)$ [4]:

$$\begin{bmatrix} \Delta T(z,1) \\ \Delta T(z,2) \\ \vdots \\ \vdots \\ \vdots \\ \Delta T(z,F) \end{bmatrix} = \begin{bmatrix} \Delta u(z,1) & 0 & \dots & \dots & \dots & 0 \\ \Delta u(z,2) & \Delta u(z,1) & \dots & \dots & \dots & 0 \\ \Delta u(z,3) & \Delta u(z,2) & \ddots & . & . & 0 \\ & & \ddots & \ddots & . & \vdots \\ & & . & \ddots & \ddots & \vdots \\ \Delta u(z,F) & \Delta u(z,F-1) & \dots & \dots & . & \Delta u(z,1) \end{bmatrix} . \begin{bmatrix} Q(z=0,1) \\ Q(z=0,2) \\ \vdots \\ \vdots \\ \vdots \\ Q(z=0,F) \end{bmatrix} \tag{4}$$

$$\Updownarrow$$

$$\mathbf{\Delta T} = \mathbf{X.Q} \tag{5}$$

The deconvolution procedure consists in reversing Eq.(5), i.e. expressing surface heat fluxes with measured surface heating:

$$\boldsymbol{Q} = \boldsymbol{X}^{-1}.\boldsymbol{\Delta T} \tag{6}$$

In the case of IR surface temperature deconvolution (z = 0), the problem is direct and matrix X inversion doesn't cause any problem. On the other hand, in the case of the deconvolution of the temperature measured by the thermocouple, the problem becomes inverse and X is ill conditioned (see §3.3). Clearly, it means that the matrix X is difficult to inverse because of very low terms in the diagonal.

3.3. Regularization procedure

The solution vector Q, is very sensitive to measurement errors contained in vector ΔT. In order to obtain a stable solution, we use a regularization procedure. For example, we can use the Thikonov regularization operator [5]. The regularized solution becomes:

$$\hat{Q}_{reg} = (X^t X + \gamma.R^t R)^{-1} \Delta T \tag{7}$$

- $\hat{Q}_{reg}$ is the regularized solution (an estimation of Q)
- γ is the regularization parameter

- R is the regularization operator depending on the type of information that we want to obtain.

In our case, we want a solution with a minimal norm of the solution (0 order) $\left\|\hat{Q}_{reg}\right\|$, so we will take R = I_d. An optimal value of the regularization parameter can be found using the "L curve" technique [6]. This type of representation allows to choose the best compromise - which is situated at the bending point of the 'L-curve' - between a stable solution, with a low value of $\left\|R\hat{Q}_{reg}\right\|$ and an accurate solution, with low residuals $\left\|X\hat{Q}_{reg} - \Delta T\right\|$. For lower values of γ (Figure 9), the solution is unstable with low residuals, on the other and, for strong values of γ (Figure 10), the solution is stable but moves away from the exact solution. On figure 11 is presented the heat estimation for a best value of γ.

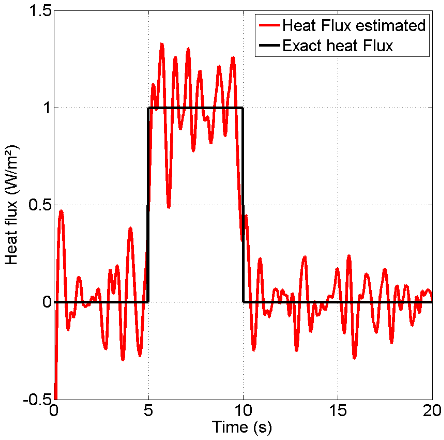

Figure 9. Heat flux estimation with a low γ

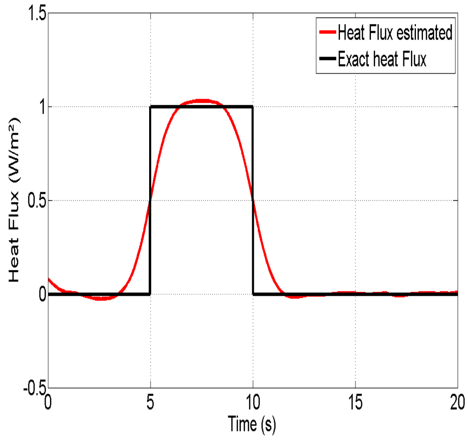

Figure 10. Heat flux estimation with a strong γ.

As example, in Figure 12 is presented the L curve and the best γ for flux estimation with the embedded measurement. One can note that the value γ depends on the level of the noise, the temporal resolution and the depth of the measurement.

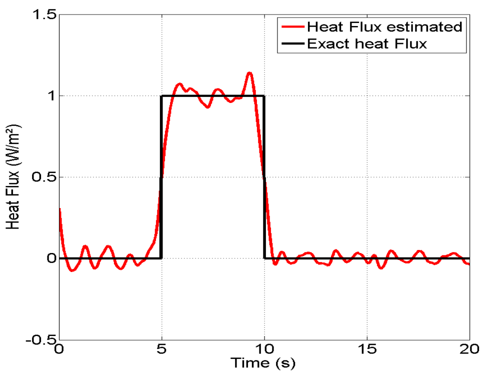

Figure 11. Heat flux estimation with the best compromise of γ.

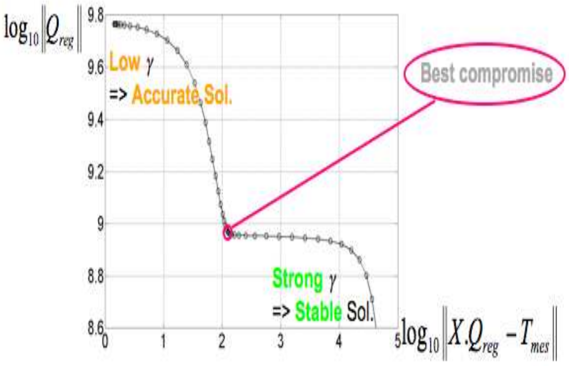

Figure 12. L curve and best γ in the case of an embedded thermocouple located at 3cm of the surface.

4. Application to a 2D experimental case

On the tile of the divertor, the heat flux depends mainly on the x location and it is deposited directly at the surface (z = 0). Furthermore the heat flux presents symmetry in the toroïdal direction. This direction is neglected. The method described above can also be applied in a 2D case but we need information on the spatial shape of the heat flux. Indeed, it's impossible to estimate simultaneously the value and the spatial shape of the heat flux with only one embedded temperature measurement. In our case, it's possible to obtain the normalised spatial shape of the heat flux since this shape is depending on the magnetic parameters of the plasma scenario. We assume that the spatial shape is not depending on time. So the heat flux function *Q(x,t)* can be decomposed (Figure 13) on a spatial function *f(x)* and temporal function *g(t).* The function *f(x)* is given by a diagnostic of magnetic lines computation, the function *g(t)* is estimating with an inversion of the thermocouple measurement.

Then, the bi-dimensional step response of the tile can be computed, for example, with the quadrupoles [7] at the thermocouple location (z = 1 cm) for a specific spatial shape (Figure 14). A regularization procedure is also applied in order to inverse the new *X* matrix.

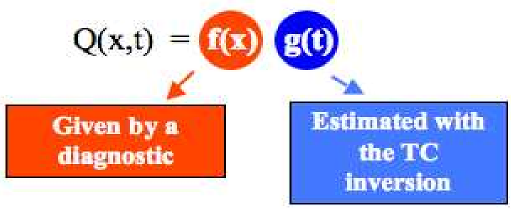

Figure 13. Temporal and spatial decomposition of the heat flux function.

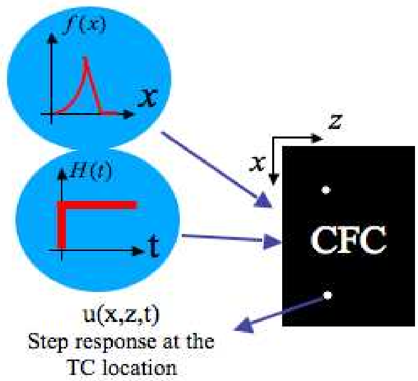

Figure 14. 2D Step response computation.

Using this method, we obtain the following results for the shot 58850 [8], the heat flux estimation is presented on figure 17. In this shot, 4 heat flux step between 1 and 6 MW/m^2 are imposed on the tile n°7 of the JET divertor. Th e heat flux spatial shape is presented on figure 16 and the TC measurement are in figure 15.

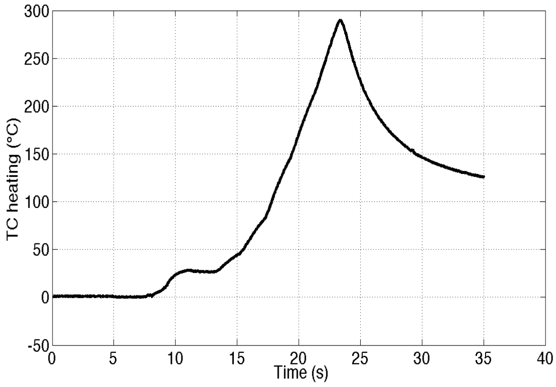

Figure 15. TC heating measurement (z = 1 cm)

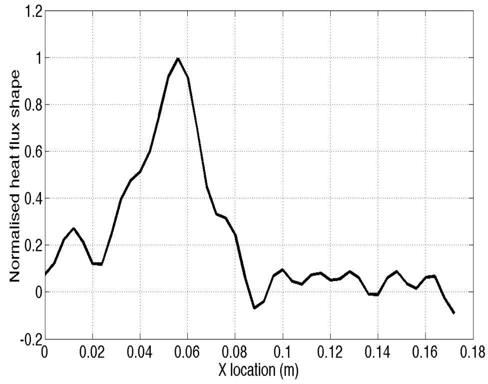

Figure 16. Heat flux spatial Shape for the shot 58850.

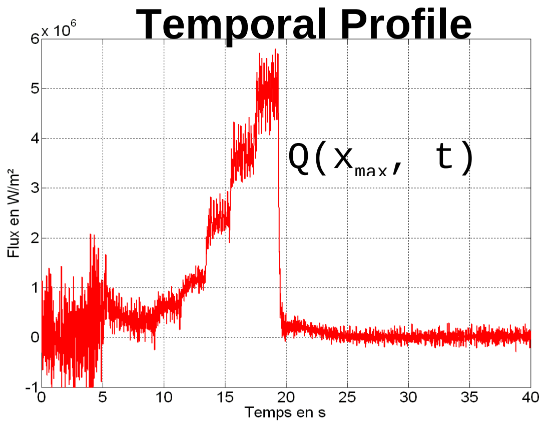

Figure 17. Heat flux estimation for the shot 58850.

5. Conclusion

The deconvolution associated to a regularization procedure can be applied in a lot of cases. We have shown during the presentation that the problem of the heat flux estimation with a thermocouple measurement in a semi-infinite wall can be solved with an Excel file. In this paper, the case of a 1D finite wall is explained. The method can be applied for others cases more complicated (multi-dimensional, multi layers, with volume sources…) if we are able to compute the step response of the material with a semi-analytical or a numerical modeling. But, the method is limited to the linear cases; it means that the thermal properties of the material are constant respect to the temperature.

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