Laboratoire d'Energétique et de Mécanique
Théorique et Appliquée

METTI 5
Mesures en Thermique et Techniques Inverses
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Tutorial T9 – 1h30

Thermal characterization of an insulating material through a three-layers transient method (experimental)

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Plan of the presentation

I. **Introduction**
II. **The classical approach**
    1. Example : the flash method
    2. Example : the hot plate method
III. **The three-layers device method**
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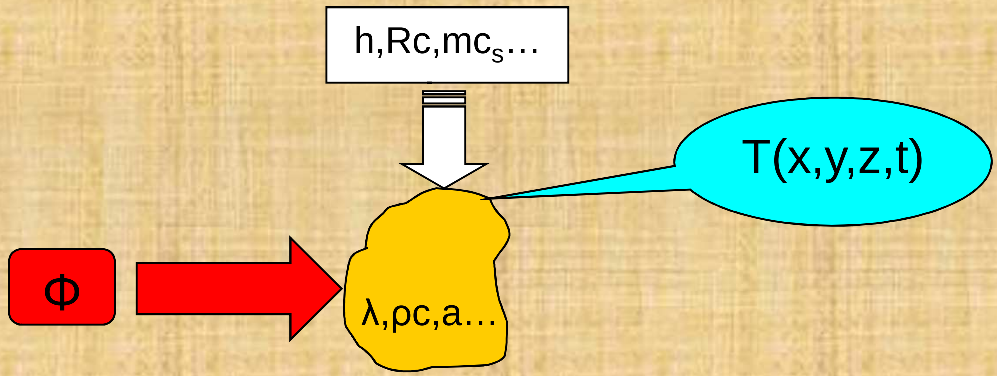
    2. Principle
    3. Modeling
    4. Sensitivity analysis
    5. Numerical simulations
IV. **Experimental part**
    1. The device
    2. Materials
    3. Measurements and data processing
V. **Conclusion**

I. Introduction

➢ Classical approach in thermal characterization



• Model under the form of a heat flow / temperature relationship

$$T(x,y,z,t) = f\left(\Phi, \lambda, \rho c, h, Rc, mc_S \ldots\right)$$

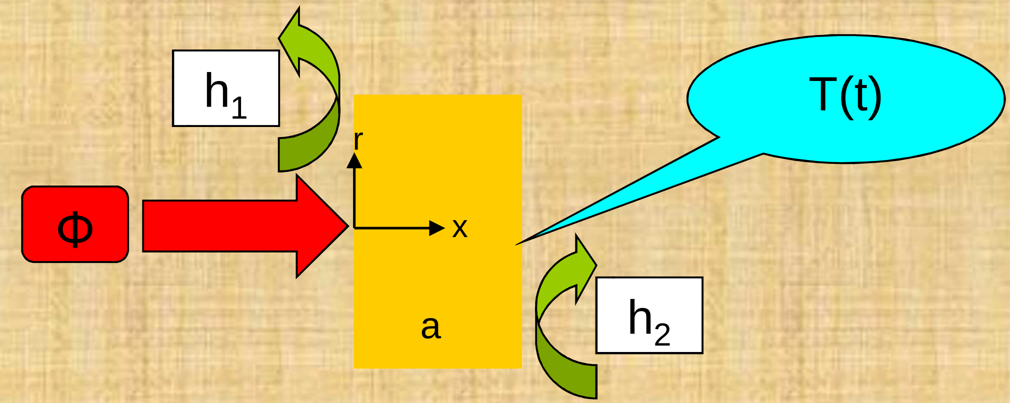
• The estimation is performed on the whole unknown parameters with two risks:
- ✓ The accuracy is decreasing with the number of parameters (correlations, local minimums)
- ✓ Some parameters are not easily quantifiable: hypothesis may introduce bias in the model

• What if the material is very insulating, has a low density or a high porosity ?
- ➢ A qualitative answer through 2 examples : the flash method and the hot plate method

• How does the proposed three-layers device method avoid these drawbacks ?
- • The technical specificity
- • The original modeling approach : a temperature / temperature relationship

II. Classical approach

1. Example: the flash method



• Model under the form:

$$T(t) = f(\Phi, a, h_1, h_2)$$

• Assuming the hypothesis that:

$$h = h_1 = h_2 = cst$$

• The following parameters are estimated: ***Φ*, *a* and *h*.**

• In case of an insulating material,

- $T_{heated\ surface} >> T$ and thus $h_1 \neq h_2$
- $T_{heated\ surface}$ has to be very high to have a good signal T
- The more insulating the material is, the more the hypothesis is contestable.

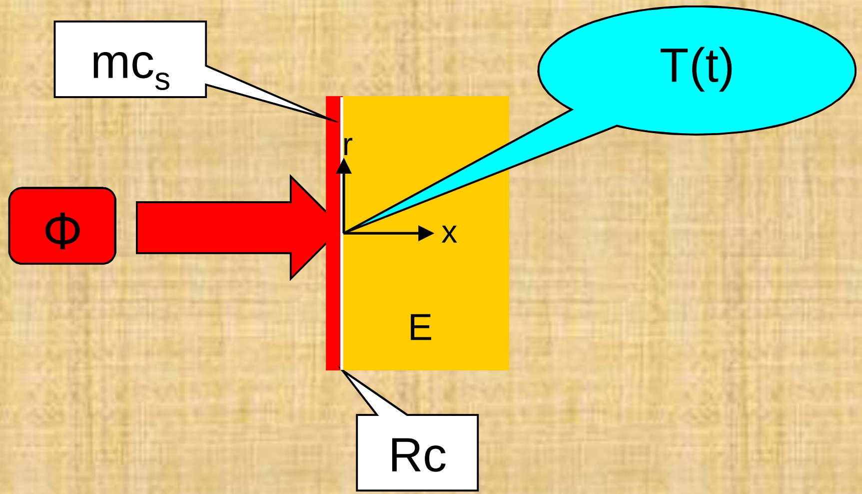
- If h_1 and h_2 are estimated separately, h_1, Φ and a get correlated between them.

• In case of a low density material

- The measurement of the surface temperature is difficult
- The heat flow may not be absorbed only at the surface but inside the sample (volume absorbtion)

II. Classical approach

2. Example: the hot plate method



• Model under the form:

$$T(t) = f(\Phi, E, Rc, mc_s)$$

• Assuming the following hypothesis:
  - Uniform contact resistance
  - Homogeneous heating element
  - Thin heating element (capacity)

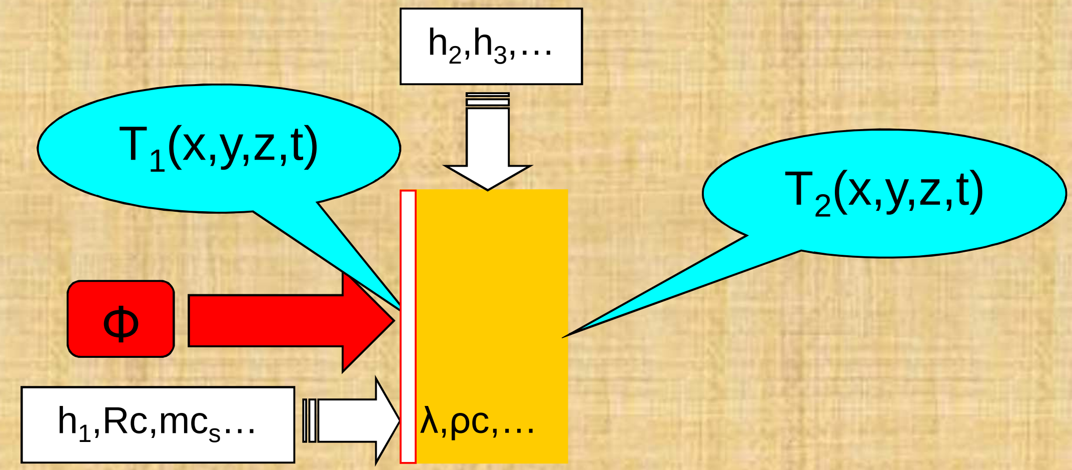
• The estimated parameters are: **E, Rc and mc_S.**

• In case of a low density material
  - ➢ The thermal capacity of the heating element is no more negligible compared to the capacity of the sample
  - ➢ The effusivity may be correlated with the thermal capacity of the heating element

• In case of an insulating material
  - ➢ The transfer into the heating element (parallel to the contact surface) is not negligible
  - ➢ To limit bounds effects, large sample are needed

III. The three-layers method

1. Proposed approach: temperature / temperature relationship



➢ Input / Output representation where the system (sample) is characterized by its transfer function H(p):

$$H(p) = \frac{L[T_2(t)]}{L[T_1(t)]}$$

• Time dependent model under the form of a convolution product:

$$T_2(t) = F(t, \lambda, \rho c, a, h_2, h_3) \otimes T_1(t)$$

Instead of: $T_2(t) = F(t, \phi, \lambda, \rho c, a, h_1, h_2, h_3, Rc, mc_S)$

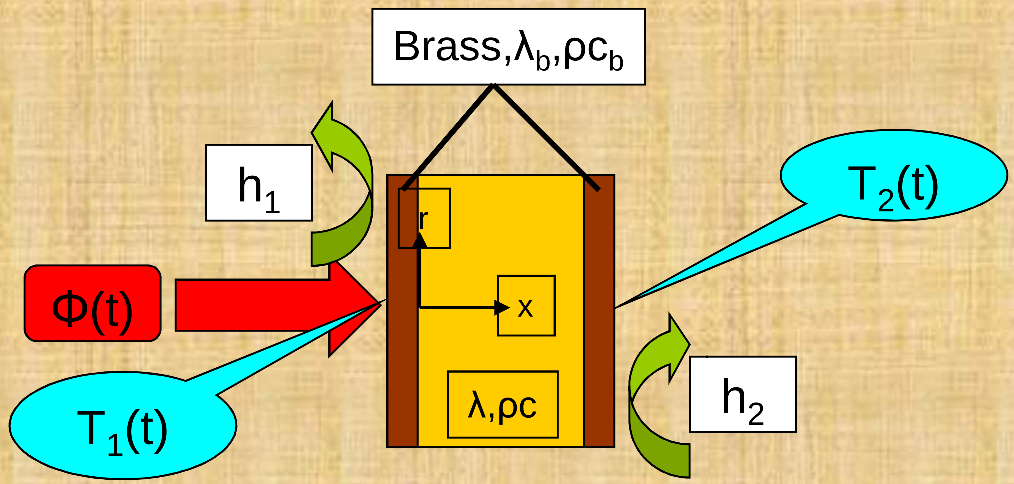
➢ **As a consequence: the model is independent of all the parameters before the input T_1: Φ,h_1,mc_S,Rc…**

III. The three-layers method

2. Principle

• Flash method = surface temperature measurement impossible for low density materials

➢ Technical solution: flash method on a three-layers system



✓ Brass discs: thin, high thermal conductivity so their temperature **T_1 and T_2 are uniform**

• The problem of the correlation between h_1, Φ and λ remains if a standard model is considered
• Trying to describe the heat flow is difficult: is it a perfect pulse ? A rectangular function ?

➢ Mathematical solution: to measure $T_1(t)$ and to use it as known data in a convolution model

$$T_2(t) = L^{-1}\left[H\left(p,\lambda,\rho c,h_2,h_3\right)\right] \otimes T_1(t)$$

III. The three-layers method

3. Modeling

Hypothesis

- 3D heat transfers with cylindrical symmetry: T=T(r,x,t)
- Contact resistances between the brass discs and the sample are neglected (insulating sample)
- Isotropic and macroscopically homogeneous sample
- Optically thick sample, no radiative transfers at the considered temperatures (ambient)
- Uniform brass discs temperature
- The convective and radiative exchange with the ambient medium are considered in a constant *h*
- The system is at thermal equilibrium at initial time

III. The three-layers method

3. Modeling

Energy conservation equation for a single layer system

$$\frac{1}{a}\frac{\partial T(r,x,t)}{\partial t}=\frac{\partial^2 T(r,x,t)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,x,t)}{\partial r}+\frac{\partial^2 T(r,x,t)}{\partial x^2}$$

$$-\lambda\left.\frac{\partial T(r,x,t)}{\partial x}\right|_{x=0}=-h_1\left(T(r,0,t)-T_\infty\right)+\varphi(t)$$

$$-\lambda\left.\frac{\partial T(r,x,t)}{\partial x}\right|_{x=e}=h_2\left(T(r,e,t)-T_\infty\right)$$

$$-\lambda\left.\frac{\partial T(r,x,t)}{\partial r}\right|_{r=0}=0$$

$$-\lambda\left.\frac{\partial T(r,x,t)}{\partial r}\right|_{r=R}=h_3\left(T(R,x,t)-T_\infty\right)$$

$$T(r,x,0)=T_\infty$$

Laplace transform

$$\theta(x,p)=L\left[T(x,t)-T_\infty\right]$$

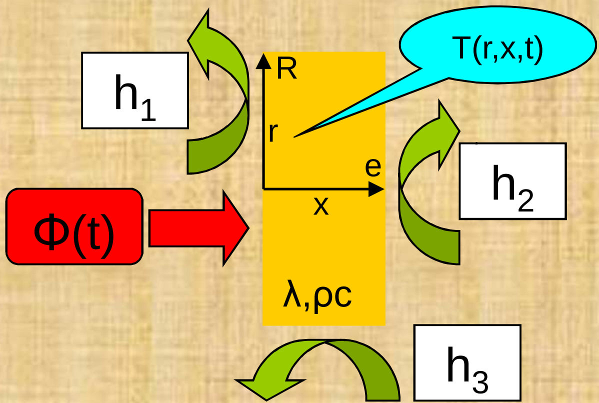
$$\frac{p}{a}\theta(r,x,p)=\frac{\partial^2\theta(r,x,p)}{\partial r^2}+\frac{1}{r}\frac{\partial\theta(r,x,p)}{\partial r}+\frac{\partial^2\theta(r,x,p)}{\partial x^2}$$

$$-\lambda\left.\frac{\partial\theta(r,x,p)}{\partial x}\right|_{x=0}=-h_1\theta(r,0,p)+\Phi(p)$$

$$-\lambda\left.\frac{\partial\theta(r,x,p)}{\partial x}\right|_{x=e}=h_2\theta(r,e,p)$$

$$\left.\frac{\partial\theta(r,x,p)}{\partial r}\right|_{r=0}=0$$

$$-\lambda\left.\frac{\partial\theta(r,x,p)}{\partial r}\right|_{r=R}=h_3\theta(R,x,p)$$



III. The three-layers method

3. Modeling

Solution of the single layer system

Integral transform + separataion of the variables:

$$\theta(r,x,p) = X(x,p)R(r,p)$$

A system of two differential equations to solve:

$$\begin{cases} \dfrac{1}{R(r,p)}\left(\dfrac{\partial^2 R(r,p)}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial R(r,p)}{\partial r}\right) = -\alpha^2 \\ \dfrac{1}{X(x,p)}\dfrac{\partial^2 X(x,p)}{\partial x^2} - \gamma^2 = 0 \end{cases}$$

Solutions for *x=0* and *x=e* as a function of *r*:

$$\theta(r,0,p) = \sum_{n=1}^{n=\infty} F_n J_0(\alpha_n r)\left[H_2 sh(\beta_n) + \beta_n ch(\beta_n)\right]$$

$$\theta(r,e,p) = \sum_{n=1}^{n=\infty} F_n J_0(\alpha_n r)\beta_n$$

Solutions for *x=0* and *x=e* averaged on *r*:

$$\theta_{moy}(0,p) = \sum_{n=0}^{n=\infty} \frac{2}{\omega_n} F_n \beta_n J_1(\omega_n)$$

$$\theta_{moy}(e,p) = \sum_{n=0}^{n=\infty} \frac{2}{\omega_n} F_n \left[\beta_n ch(\beta_n) + H_2 sh(\beta_n)\right] J_1(\omega_n)$$

III. The three-layers method

3. Modeling

Including the brass discs

Brass layers heat balance

$$-\lambda\frac{\partial\theta_{isomoy}(x,p)}{\partial x}=-\left(e_1\rho c_1 p+\left[h_1+\frac{2e_1h_3}{R}\right]\right)\theta_1(p)+\Phi(p)$$

$$-\lambda\frac{\partial\theta_{isomoy}(x,p)}{\partial x}=\left(e_2\rho c_2 p+\left[h_2+\frac{2e_2h_3}{R}\right]\right)\theta_2(p)$$

Single layer limit conditions

$$-\lambda\left.\frac{\partial\theta(r,x,p)}{\partial x}\right|_{x=0}=-h_1\theta(r,0,p)+\Phi(p)$$

$$-\lambda\left.\frac{\partial\theta(r,x,p)}{\partial x}\right|_{x=e}=h_2\theta(r,e,p)$$

- Considering the average value of the temperature on the variable r, the brass layers heat balance is similar to the limit conditions at *x=0* and *x=e* of the single layer system (the sample)

- The brass discs are taken into account in the single layer solution by introducing the modified coefficients:

$$h_1^*=e_1\rho c_1 p+\left[h_1+\frac{2e_1h_3}{R}\right]\qquad h_2^*=e_2\rho c_2 p+\left[h_2+\frac{2e_2h_3}{R}\right]$$

III. The three-layers method

3. Modeling

Solution of the three-layers system

The front and rear brass discs temperature:

$$\theta_{moy}(0,p)=\sum_{n=1}^{n=\infty}\frac{4\Phi(p)\frac{e}{\lambda}\left[\beta_n ch(\beta_n)+H_2 sh(\beta_n)\right]}{\omega_n^2\left(1+\frac{\omega_n^2}{H_3^2}\right)\left[sh(\beta_n)\left[\beta_n^2+H_1H_2\right]+ch(\beta_n)\left[\beta_n(H_2+H_1)\right]\right]}$$

$$\theta_{moy}(e,p)=\sum_{n=1}^{n=\infty}\frac{4\Phi(p)\frac{e}{\lambda}\beta_n}{\omega_n^2\left(1+\frac{\omega_n^2}{H_3^2}\right)\left[sh(\beta_n)\left[\beta_n^2+H_1H_2\right]+ch(\beta_n)\left[\beta_n(H_2+H_1)\right]\right]}$$

With ω_n which are the N first solutions of the transcendent equation: $\omega J_1(\omega)=H_3 J_0(\omega)$

And: $\beta_n=\sqrt{\frac{pe^2}{a}+\left(\frac{e\omega_n}{R}\right)^2}$ $H_1=\frac{h_1^* e}{\lambda}$ $H_2=\frac{h_2^* e}{\lambda}$ $H_3=\frac{h_3 R}{\lambda}$

III. The three-layers method

3. Modeling

The convolution model

Considering the transfer function: $H(p) = \dfrac{\theta_{moy}(e, p)}{\theta_{moy}(0, p)}$

The rear brass disc temperature: $\theta_2(p) = H(p, \lambda, \rho c, h_2, h_3) \cdot \theta_1(p)$

The time dependent expression of the rear brass disc temperature: $T_2(t) = L^{-1}[H(p, \lambda, \rho c, h)] \otimes T_1(t)$

T_1 is assumed to be a known parameter of the model and is measured: $T_1(t) = T_{1\exp}(t)$

The direct model explains $T_2(t)$ as: $T_2(t) = L^{-1}[H(p, \lambda, \rho c, h)] \otimes T_{1\exp}(t)$

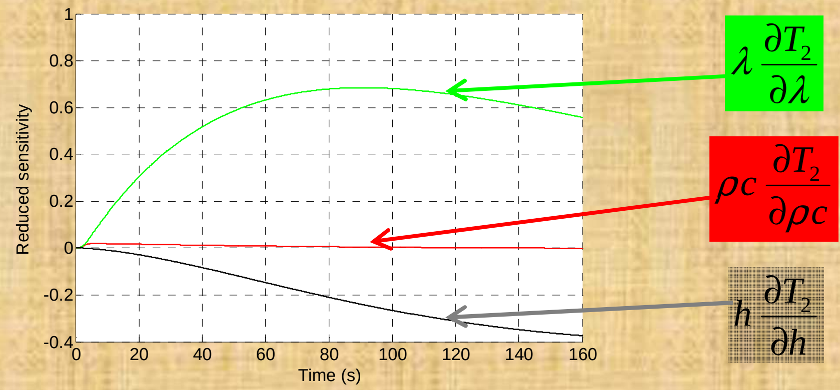
The remaining unknown parameters are λ, ρc and h which are estimated minimizing: $\sum [T_2(t) - T_{2\exp}(t)]^2$

- **h_1 and the properties of the front brass disc have no effect on T_2**
- **No hypothesis about the heat flow (space and time)**
- **3 parameters to estimate**

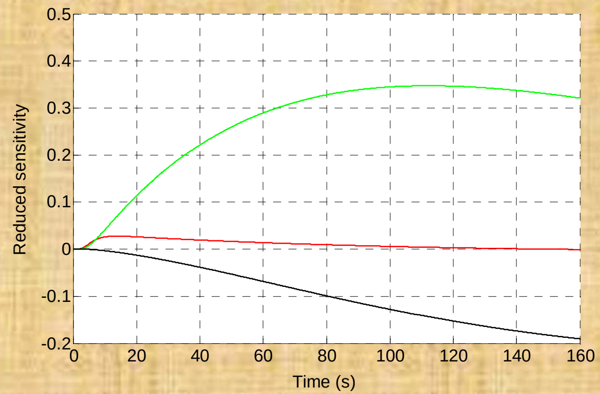
III. The three-layers method

4. Sensitivity analysis

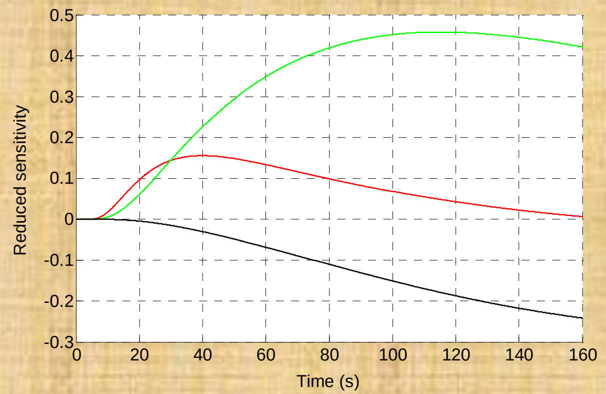
Super-insulating material: *e* = 5mm
$\lambda = 0.02\ W.m^{-1}.K^{-1}$, $\rho c = 5\ 000\ J.m^{-3}.K^{-1}$



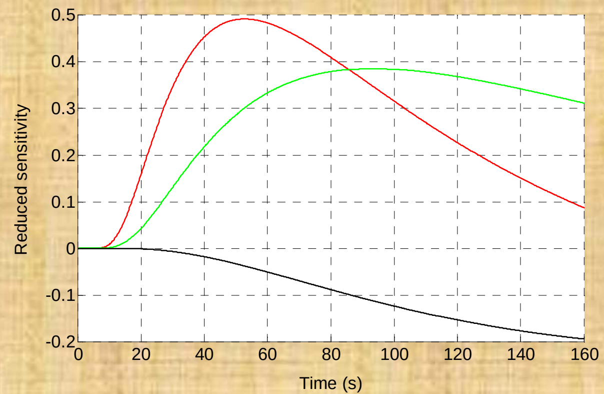
Super-insulating material: *e* = 10mm
$\lambda = 0.02\ W.m^{-1}.K^{-1}$, $\rho c = 5\ 000\ J.m^{-3}.K^{-1}$



Polystyrene: *e* = 10mm
$\lambda = 0.035\ W.m^{-1}.K^{-1}$, $\rho c = 45\ 000\ J.m^{-3}.K^{-1}$



Cellular concrete: *e* = 10mm
$\lambda = 0.15\ W.m^{-1}.K^{-1}$, $\rho c = 300\ 000\ J.m^{-3}.K^{-1}$



III. The three-layers method

4. Numerical simulations

Thermograms T_{1exp} and T_{2exp} computed with COMSOL considering the following nominal data

	Diameter (mm)	Thickness (mm)	λ ($W.m^{-1}.K^{-1}$)	ρc ($J.m^{-3}.K^{-1}$)	a ($m^2.s^{-1}$)
Sample	35	5.6	0.02	5 000	4×10^{-6}
Metallic discs (copper)	35	0.4	397.5	3 440 000	1.15×10^{-4}

✓ The hypothesis of uniforms temperature in the metallic discs is valid (COMSOL)

Two estimation methods have been applied on these obtained thermograms:

- The heat flow dependent model: Only T_{2exp} has been considered as experimental data and $\Phi, \lambda, \rho c$ and h have been estimated using a least squares algorithm

- The front temperature dependent model: T_{1exp} has been considered as a time dependent known parameter and $\lambda, \rho c$ and h have been estimated using a least squares algorithm on T_{2exp}

III. The three-layers method

4. Numerical simulations

Results for different cases of exchange coefficient h_1, h_2 and h_3 set in COMSOL

	λ ($W.m^{-1}.K^{-1}$)	ρc ($J.m^{-3}.K^{-1}$)	h ($W.m^{-1}.K^{-1}$)
Nominal value	0.0200	5 000	$h_1=h_2=h_3=10$
Heat flow model	0.0165	3 929	11.3
Front temp. model	0.0199	5 237	11.2

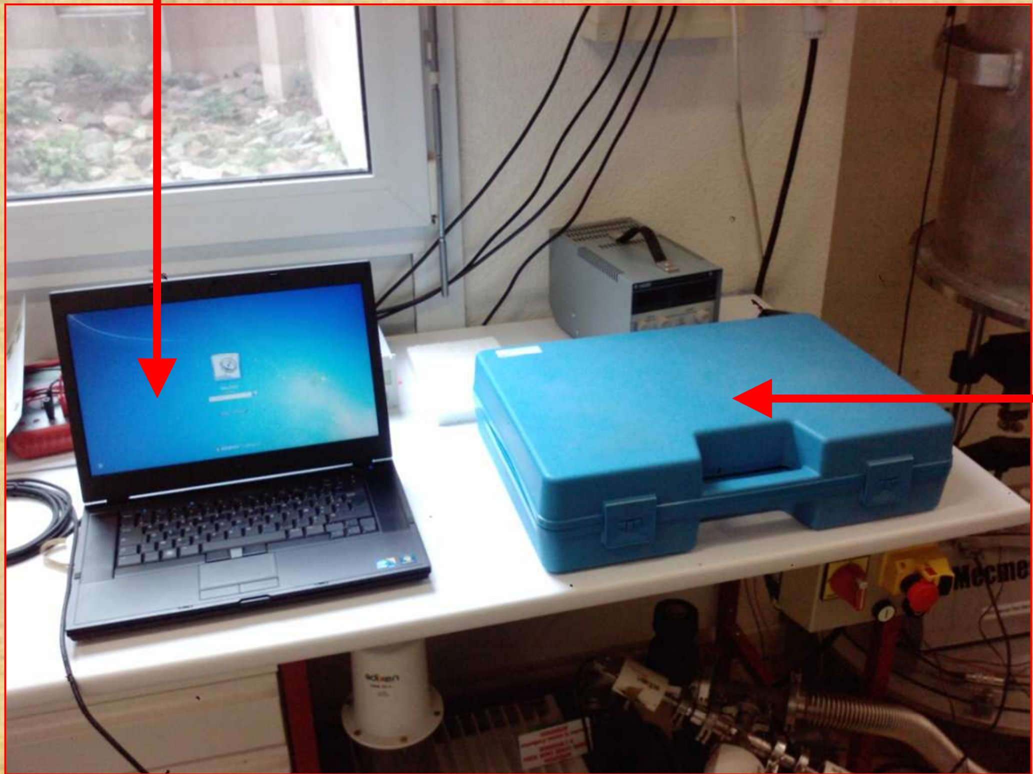
	λ ($W.m^{-1}.K^{-1}$)	ρc ($J.m^{-3}.K^{-1}$)	h ($W.m^{-1}.K^{-1}$)
Nominal value	0.0200	5 000	$h_1=15$ $h_2=h_3=5$
Heat flow model	0.0340	8 293	7.5
Front temp. model	0.0199	5 103	5.5

- The heat flow dependent model is not precise and is very sensible to h_1.
- The front temperature dependent model gives very good results and is not sensible to the conditions on the front surface (heat flow, heat losses) as announced before
- In addition, it has been verified that errors on thermal conductivity of the metallic discs have no consequences and an error of x% on thermal capacity of the disc induces the same error on the thermal conductivity of the sample

IV. Measurements

1. The device

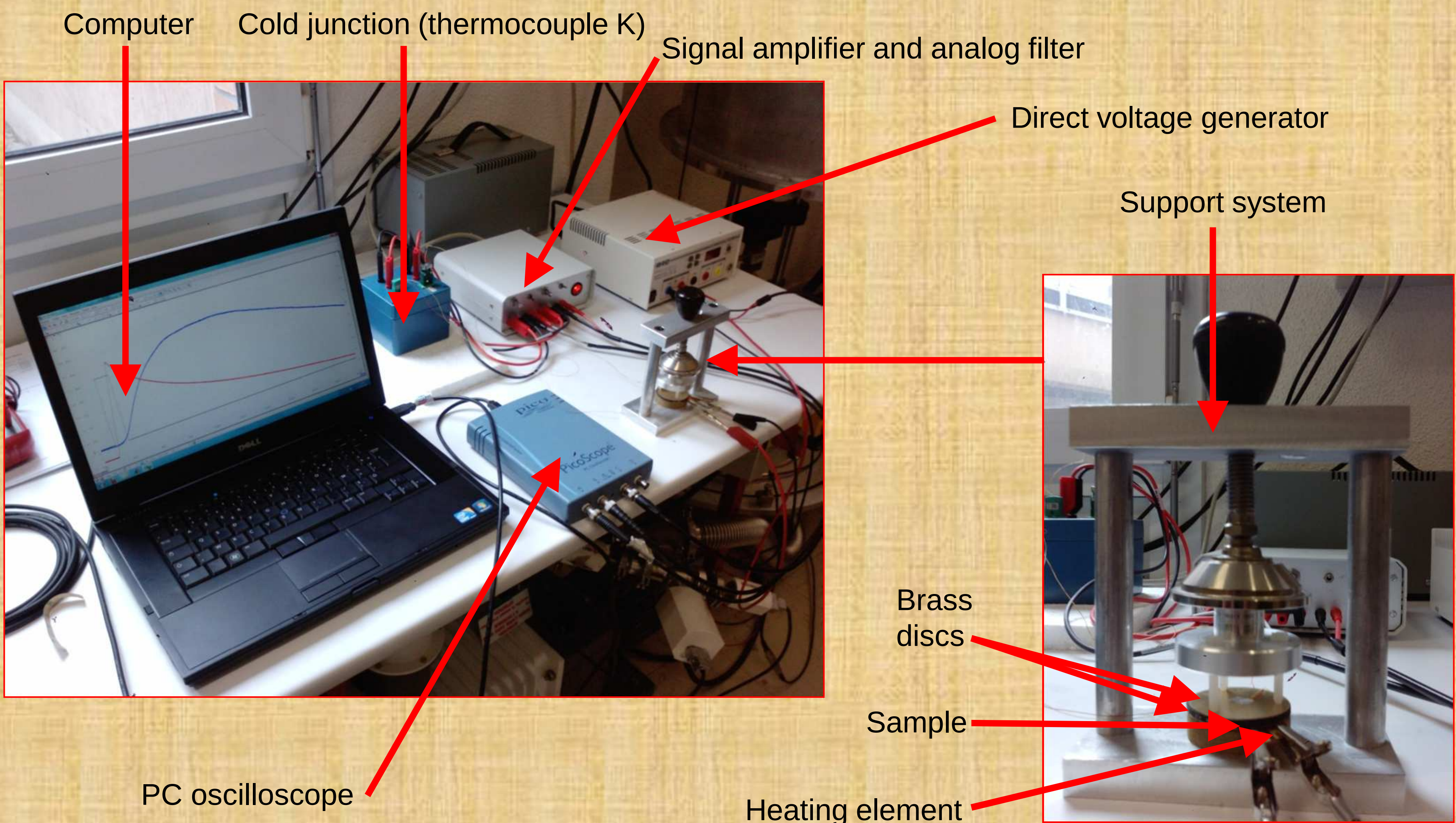
Computer



Case

IV. Measurements

1. The device



IV. Measurements

2. Materials

Proposed samples

Material	D (mm)	e (mm)	ρ (kg.m^{-3})	c (J.kg^{-1}.K^{-1})	ρc (J.m^{-3}.K^{-1})
Polyethylene foam	40	6.40	40	2 061	82 440
Depron (extruded polystyrene)	40	5.65	35	1 039	36 365
Spaceloft (silica aerogel)	40	2.75	145	1 095	158 775
PVC	40	2.95	1450	952	1 380 000

Metallic discs

Material	D (mm)	e (mm)	ρ (kg.m^{-3})	c (J.kg^{-1}.K^{-1})	ρc (J.m^{-3}.K^{-1})
Polished brass	40	0.38	8 400	377	3 166 800

IV. Measurements

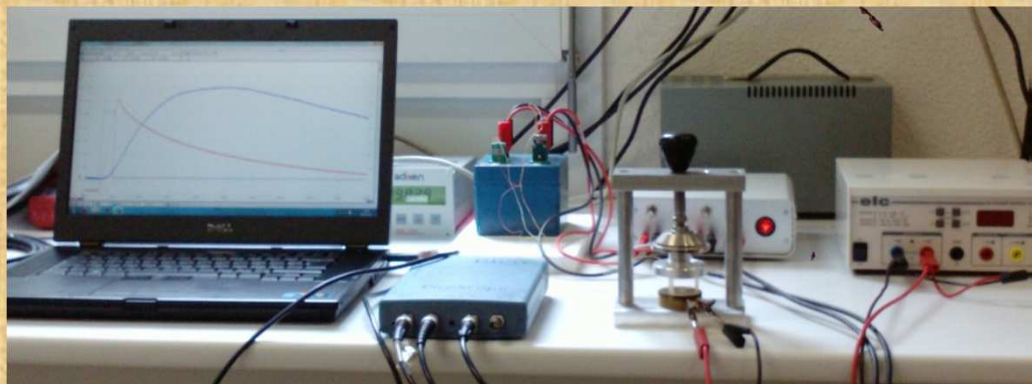
3. Measurements and data processing

Real time experiment will be carried out on one of our proposed materials

- PicoScope 6 software for the temperature signals acquisition
- Matlab for data processing

Pre-recorded data will be processed on the other materials

- Matlab for the data processing
- Our mind for discussion, questions and eventually answers



IV. Conclusions

3. Measurements and data processing

Classical approach

- Limits in extreme cases : Very light and insulating materials

Advantages of the approach “temperature / temperature” relationship via a convolution model

- Low number of unknown parameters
- No hypothesis about the front surface exchange
- No information about the spatial and temporal forms of the heat flow needed

Advantages of the specific device “three-layers” using this approach

- Surface temperature measurement reliable
- Suited to small samples
- Estimation method is robust and the parameters are decorrelated

Limits of the three-layers method

- Not optimal for heavy materials
- Estimation of the heat capacity less precise

End of the tutorial

Thank you for your participation !