Centre de Thermique de Lyon

Tutorial 8

Analysis of Error Factors for Measurement Data and Inverse Techniques Application to Temperature Measurements and Heat Flux Estimations

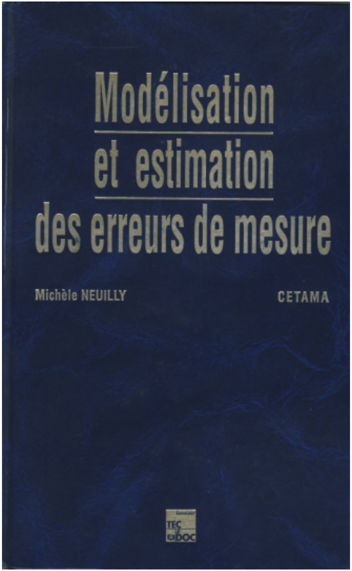
Damien DAVID

Centre de Thermique de Lyon
INSA de Lyon - CNRS - UCBL

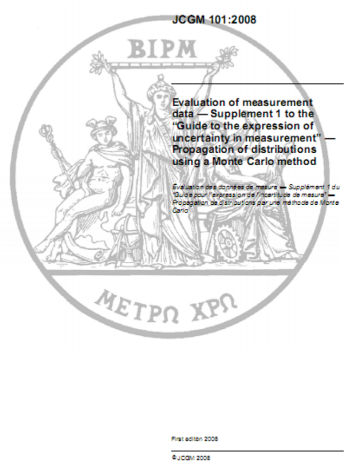
14th June 2011

Content of the section

Bibliography



Bibliography



JCGM 101:2008

BIPM

Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — Propagation of distributions using a Monte Carlo method

Évaluation des données de mesure — Supplément 1 du "Guide pour l'expression de l'incertitude de mesure" — Propagation de distributions par une méthode de Monte Carlo

ΜΕΤΡΩ ΧΡΩ

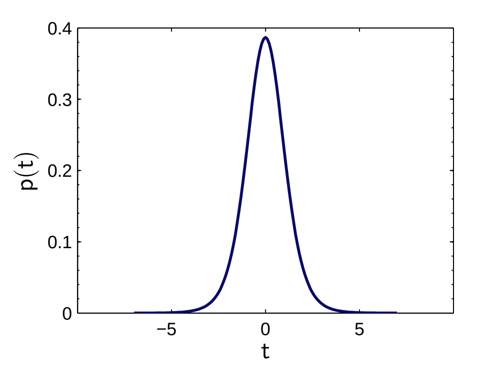
First edition 2008

© JCGM 2008

Definitions : Statistics

t-Distribution Random Variable

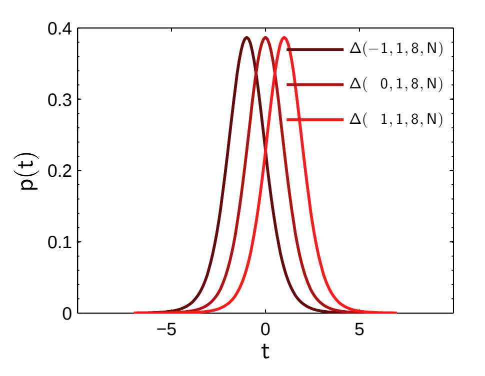
$$\Delta(E, s^2, \nu, N)$$



Definitions : Statistics

t-Distribution Random Variable

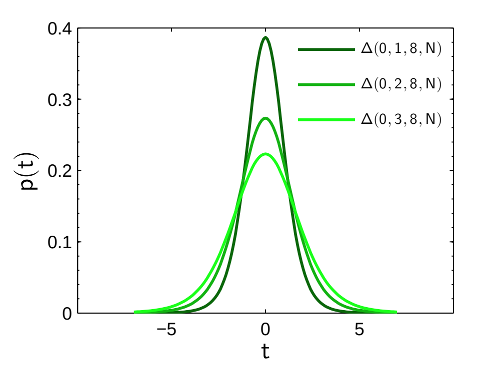
$\Delta(E, s^2, \nu, N)$ Mathematical Expectation



Definitions : Statistics

t-Distribution Random Variable

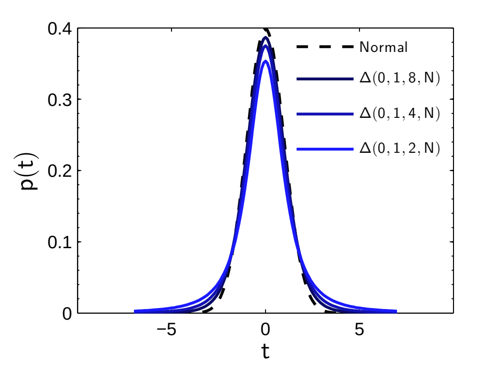
$\Delta(\ E\ ,\ s^2\ ,\ \nu\ ,\ N\ )$ Variance



Definitions : Statistics

t-Distribution Random Variable

$\Delta(E , s^2 , \nu , N)$ Degree of Freedom



Definitions : Statistics

t-Distribution Random Variable

$\Delta(E, s^2, \nu, N)$ Number of associated elements

Definitions : Statistics

t-Distribution Random Variable

$\Delta(E, s^2, \nu, N)$

Interval of confidence :

$\Delta^{95\%} = E \pm t_{\nu}^{0.975}\sqrt{s^2}$

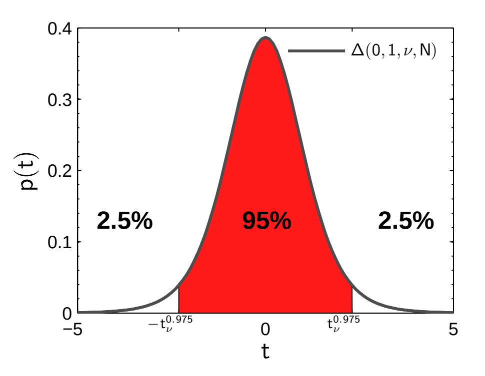
Definitions : Statistics

t-Distribution Random Variable

$\Delta(E, s^2, \nu, N)$

Interval of confidence :

$$\Delta^{95\%} = E \pm t_{\nu}^{0.975}\sqrt{s^2}$$



Definitions

Estimator of Y_{true}

Measurement Device	Data Treatment
(Thermocouple, Balance...)	(Averaging)

Definitions

Estimator of Y_{true}

Measurement Device	Data Treatment
(Thermocouple, Balance...)	(Averaging)
Raw Data	Estimation

$$Y_m^p$$

$$\overline{Y}_m = \frac{\sum_{p=1}^{N_m} Y_m^p}{N_m}$$

Definitions

Estimator of Y_{true}

Measurement Device	Data Treatment
(Thermocouple, Balance...)	(Averaging)
Raw Data	Estimation
Y_m^p	$\overline{Y}_m = \frac{\sum_{p=1}^{N_m} Y_m^p}{N_m}$
Raw Data Total Error	Estimation Total Error
$Y_m^p = Y_{\text{true}} + \varepsilon_{Y_m}^p$	$\overline{Y}_m = Y_{\text{true}} + \varepsilon_{\overline{Y}_m}$

Definitions

Estimator of Y_{true}

Measurement Device	Data Treatment
(Thermocouple, Balance...)	(Averaging)
Raw Data	**Estimation**
Y_m^p	$\overline{Y}_m = \dfrac{\sum_{p=1}^{N_m} Y_m^p}{N_m}$
Raw Data Total Error	**Estimation Total Error**
$Y_m^p = Y_{\text{true}} + \varepsilon_{Y_m}^p$	$\overline{Y}_m = Y_{\text{true}} + \varepsilon_{\overline{Y}_m}$
$\varepsilon_{Y_m}^p \in \mathcal{N}_Y(0, \sigma_Y)$	$\varepsilon_{\overline{Y}_m} \in \mathcal{N}_{\overline{Y}_m}(0, \sigma_{\overline{Y}_m})$

Centre de Thermique de Lyon

Definitions

Estimator of Y_{true}

Measurement Device	Data Treatment
(Thermocouple, Balance...)	(Averaging)
Raw Data	Estimation
Y_m^p	$\overline{Y}_m = \dfrac{\sum_{p=1}^{N_m} Y_m^p}{N_m}$
Raw Data Total Error	Estimation Total Error
$Y_m^p = Y_{\text{true}} + \varepsilon_{Y_m}^p$	$\overline{Y}_m = Y_{\text{true}} + \varepsilon_{\overline{Y}_m}$
$\varepsilon_{Y_m}^p \in \mathcal{N}_Y(0, \sigma_Y)$	$\varepsilon_{\overline{Y}_m} \in \mathcal{N}_{\overline{Y}_m}(0, \sigma_{\overline{Y}_m})$
Raw Data Total Error Random Variable	Estimation Total Error Random Variable
$\Delta_Y(0, s_Y^2, \nu_Y, N_Y)$	$\Delta_{\overline{Y}_m}(0, s_{\overline{Y}_m}^2, \nu_{\overline{Y}_m}, N_{\overline{Y}_m})$

Definitions

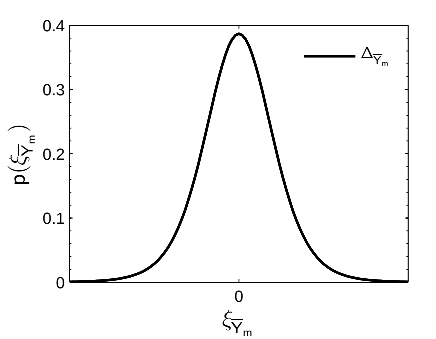
Estimator Uncertainty

$Y_{\text{true}} = \overline{Y}_m \pm \Delta^{95\%}_{\overline{Y}_m}$ is true with a level of confidence 95%

Definitions

Estimator Uncertainty

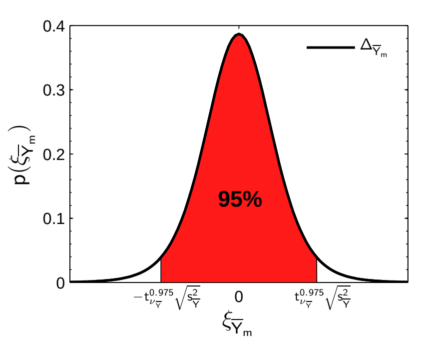
$Y_{\text{true}} = \overline{Y}_m \pm \Delta^{95\%}_{\overline{Y}_m}$ is true with a level of confidence 95%



Definitions

Estimator Uncertainty

$Y_{\text{true}} = \overline{Y}_m \pm \Delta^{95\%}_{\overline{Y}_m}$ is true with a level of confidence 95%



$$\Delta^{95\%}_{\overline{Y}_m} = t^{0.975}_{\nu_{\overline{Y}_m}} \sqrt{s^2_{\overline{Y}_m}}$$

Definitions

Error Factors f_q

Physical Phenomena which deteriorate the quality of the measurement. They must be independent !

Fixed Error Factors	Fluctuating Error Factor
Temperature, Pollutant Concentration, Air speed, measurement tool...	Electromagnetic noise...
$f_1..f_{M-1}$	f_M

Definitions

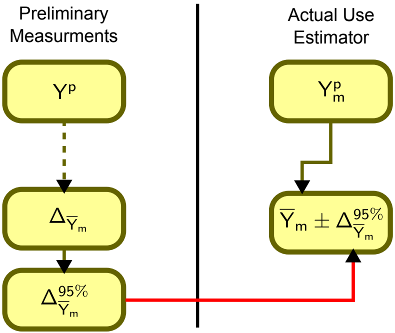
Error Factors f_q

Physical Phenomena which deteriorate the quality of the measurement. They must be independent !

Fixed Error Factors	Fluctuating Error Factor
Temperature, Pollutant Concentration, Air speed, measurement tool...	Electromagnetic noise...
$f_1..f_{M-1}$	f_M
Raw Data Total Error	**Estimation Total Error**
$Y_m^p = Y_{\text{true}} + \varepsilon^p_{Y_m}$	$\overline{Y}_m = Y_{\text{true}} + \varepsilon_{\overline{Y}_m}$
Raw Data Associated Errors	**Estimation Associated Errors**
$Y_m^p = Y_{\text{true}} + \varepsilon^p_{Y_m,f_1} + .. + \varepsilon^p_{Y_m,f_M}$	$\overline{Y}_m = Y_{\text{true}} + \varepsilon^p_{\overline{Y}_m,f_1} + .. + \varepsilon^p_{\overline{Y}_m,f_M}$

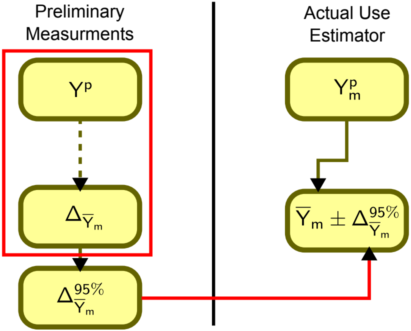
Scope

- Conservation of $\Delta_{\overline{Y}_m}$



Scope

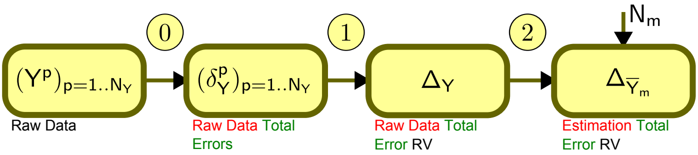
- Conservation of $\Delta_{\overline{Y}_m}$



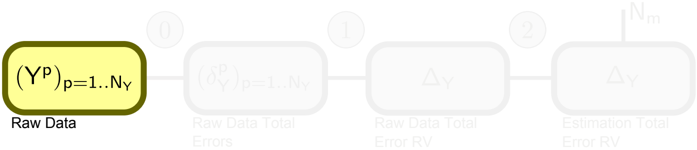
Content of the section

Centre de Thermique de Lyon

No Error Factor Analysis



No Error Factor Analysis

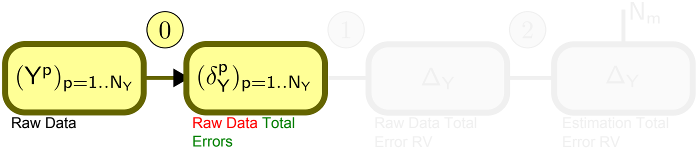


Population of raw data

- Measurement of the same true value Y_{true}
- Y_{true} not necessarily known.

$$Y^p = Y_{\text{true}} + \varepsilon_Y^p$$

No Error Factor Analysis

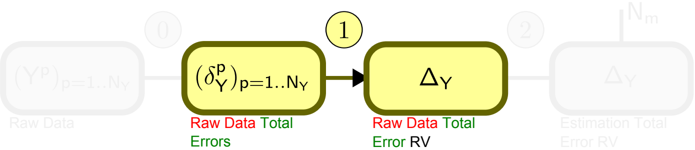


Step 0 : Approximation of the raw data errors

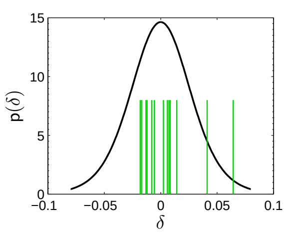
- Approximation of ε_Y^p

$$\delta_Y^p = Y^p - \overline{Y} = \varepsilon_Y^p - \overline{\varepsilon}_Y^p$$

No Error Factor Analysis



Step 1 : Determination of Δ_Y

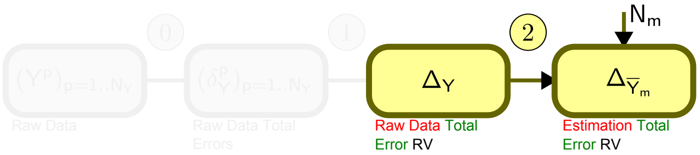


$$N_Y = N_Y$$

$$\nu_Y = N_Y - 1$$

$$s_Y^2 = \frac{\sum_{p=1}^{N_Y} (\delta_Y^p)^2}{\nu_Y}$$

No Error Factor Analysis



Step 3 : Determination of $\Delta_{\overline{Y}_m}$

$$\boxed{\frac{\varepsilon^1 + .. + \varepsilon^{N_m}}{N_m}}$$

$$N_{\overline{Y}_m} = N_Y$$

$$\nu_{\overline{Y}_m} = \nu_Y$$

$$\boxed{s^2_{\overline{Y}_m} = \frac{s^2_Y}{N_m}}$$ Decreasing

No Error Factor Analysis

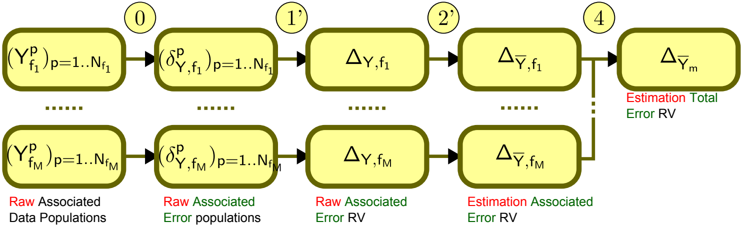
Advantages

- Quick Method
- Fluctuation of the Estimation Value

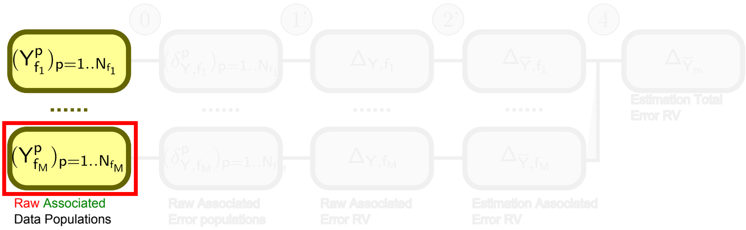
Drawbacks

- No Bias
- No Interpretation of the Error Structure

Error Factor Analysis



Error Factor Analysis

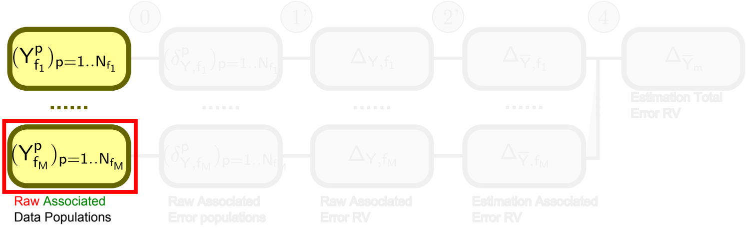


Population of raw data

- f_k are blocked by controlling environment

$$
\begin{aligned}
Y^1_{f_M} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \varepsilon^1_{Y,f_M} \\
Y^2_{f_M} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \varepsilon^2_{Y,f_M} \\
& \ldots\ldots\ldots\ldots\ldots\ldots \\
Y^{N_{f_M}}_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \varepsilon^{N_{f_M}}_{Y,f_M}
\end{aligned}
$$

Error Factor Analysis

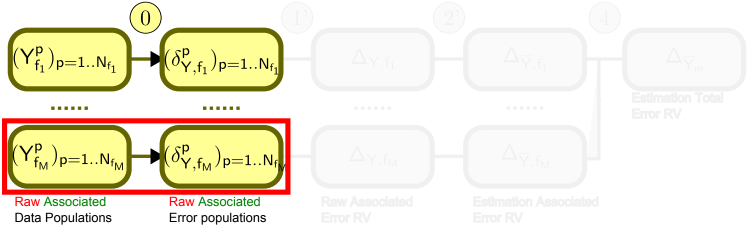


Population of raw data

- f_k are blocked by controlling environment

$$
\begin{aligned}
Y^1_{f_M} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \varepsilon^1_{Y,f_M} \\
Y^2_{f_M} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \varepsilon^2_{Y,f_M} \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
Y^{N_{f_M}}_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \varepsilon^{N_{f_M}}_{Y,f_M} \\
\hline
\overline{Y}_{f_M} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \overline{\varepsilon}_{Y,f_M}
\end{aligned}
$$

Error Factor Analysis

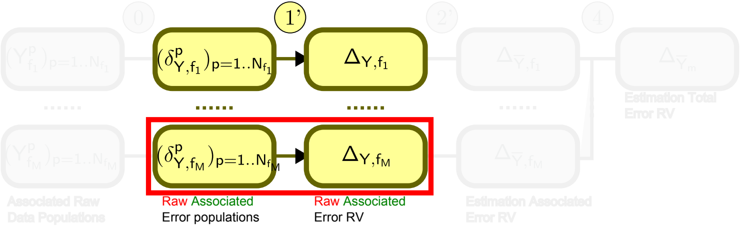


Step 0 : Approximation of the raw associated errors

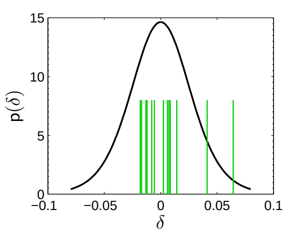
- Approximation of ε^p_{Y,f_M}

$$\delta^p_{Y,f_M} = Y^p_{f_M} - \overline{Y}_{f_M} = \varepsilon^p_{Y,f_M} - \overline{\varepsilon}_{Y,f_M}$$

Error Factor Analysis



Step 1 : Determination of Δ_{Y,f_M}

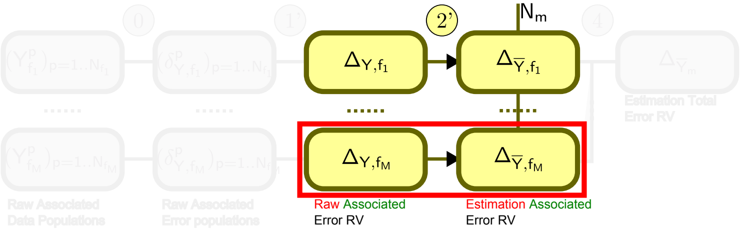


$$N_{Y,f_M} = N_{f_M}$$

$$\nu_{Y,f_M} = N_{f_M} - 1$$

$$s^2_{Y,f_M} = \frac{\sum_{p=1}^{N_{f_M}} (\delta^p_{Y,f_M})^2}{\nu_{Y,f_M}}$$

Error Factor Analysis



Step 2 : Determination of $\Delta_{\overline{Y},f_M}$

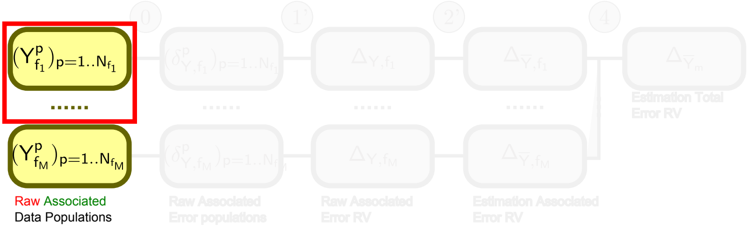
$$\boxed{\frac{\varepsilon^1 + .. + \varepsilon^{N_m}}{N_m}}$$

$$N_{\overline{Y},f_M} = N_{Y,f_M}$$

$$\nu_{\overline{Y},f_M} = \nu_{Y,f_M}$$

$$\boxed{s^2_{\overline{Y},f_M} = \frac{s^2_{Y,f_M}}{N_m}}$$ Decreasing

Error Factor Analysis

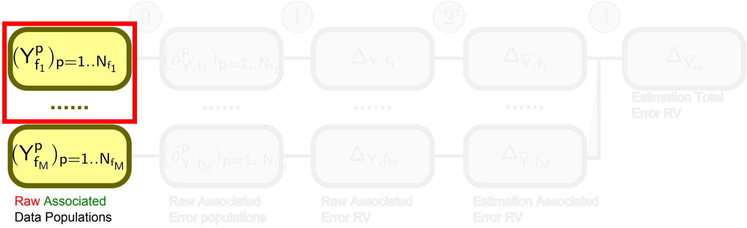


Population of raw data

- f_M blocked by averaging

$$
\begin{aligned}
Y^1_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon^1_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \overline{\varepsilon}^1_{Y,f_M} \\
Y^2_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon^2_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \overline{\varepsilon}^2_{Y,f_M} \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
Y^{N_{f_q}}_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \ldots + \varepsilon_{Y,f_{q-1}} + \varepsilon^{N_{f_q}}_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \ldots + \overline{\varepsilon}^{N_{f_q}}_{Y,f_M}
\end{aligned}
$$

Error Factor Analysis



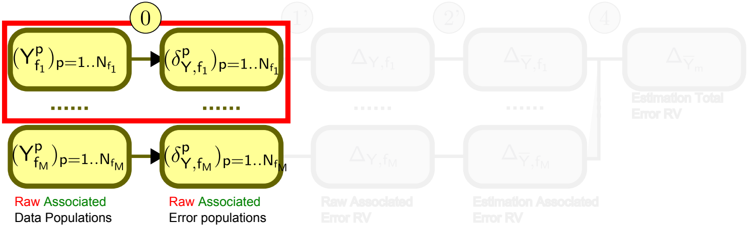
Population of raw data

- f_M blocked by averaging

$$
\begin{aligned}
Y^1_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \dots + \varepsilon_{Y,f_{q-1}} + \varepsilon^1_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \dots + \overline{\varepsilon}^1_{Y,f_M} \\
Y^2_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \dots + \varepsilon_{Y,f_{q-1}} + \varepsilon^2_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \dots + \overline{\varepsilon}^2_{Y,f_M} \\
&\dots\dots\dots\dots\dots\dots \\
Y^{N_{f_q}}_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \dots + \varepsilon_{Y,f_{q-1}} + \varepsilon^{N_{f_q}}_{Y,f_q} + \varepsilon_{Y,f_q} + \dots + \overline{\varepsilon}^{N_{f_q}}_{Y,f_M} \\
\hline
\overline{Y}_{f_q} &= Y_{\text{true}} + \varepsilon_{Y,f_1} + \dots + \varepsilon_{Y,f_{q-1}} + \overline{\varepsilon}_{Y,f_q} + \varepsilon_{Y,f_{q+1}} + \dots + \overline{\overline{\varepsilon}}_{Y,f_M}
\end{aligned}
$$

Centre de Thermique de Lyon

Error Factor Analysis

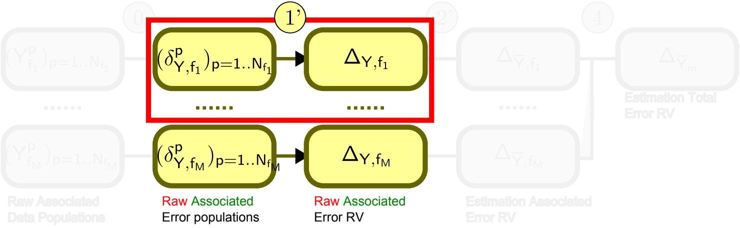


Step 0 : Approximation of the raw associated errors

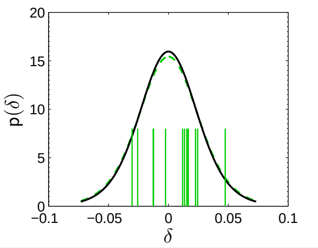
- Approximation of ε^p_{Y,f_q}

$$\delta^p_{Y,f_q} = Y^p_{f_q} - \overline{Y}_{f_q} = (\varepsilon^p_{Y,f_q} - \overline{\varepsilon}_{Y,f_q}) + (\overline{\varepsilon}^p_{Y,f_M} - \overline{\overline{\varepsilon}}_{Y,f_M})$$

Error Factor Analysis



Step 1 : Determination of Δ_{Y,f_q}



$$N_{Y,f_q} = N_{f_q}$$

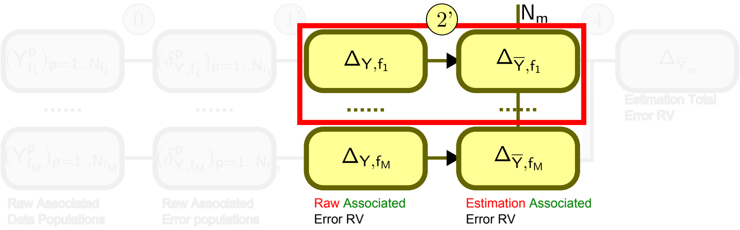
$$\nu_{Y,f_q} = N_{f_q} - 1$$

$$\boxed{s^2_{Y,f_q} = \frac{\sum_{p=1}^{N_{f_q}} (\delta^p_{Y,f_q})^2}{\nu_{Y,f_q}} - \frac{s^2_{Y,f_M}}{N_{f_q}}}$$

Compensation

Centre de Thermique de Lyon

Error Factor Analysis



Step 2 : Determination of $\Delta_{\overline{Y},f_q}$

$$\boxed{\frac{\varepsilon^1 + .. + \varepsilon^{N_m}}{N_m}}$$

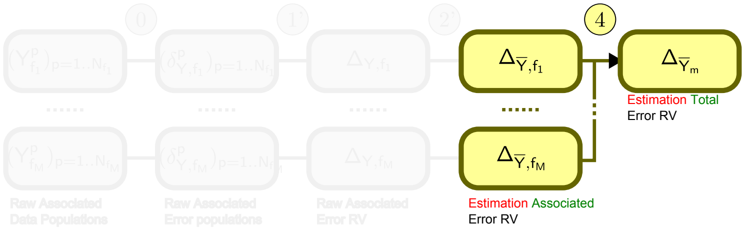
$$N_{\overline{Y},f_q} = N_{Y,f_q}$$

$$\nu_{\overline{Y},f_q} = \nu_{Y,f_q}$$

$$\boxed{s^2_{\overline{Y},f_q} = \frac{s^2_{Y,f_q}}{N_m}}$$ Fluctuating Error

$$\boxed{s^2_{\overline{Y},f_q} = s^2_{Y,f_q}}$$ Bias

Error Factor Analysis



Step 4 : Determination of $\Delta_{\overline{Y}_m}$

$$\boxed{\varepsilon_{f_1} + .. + \varepsilon_{f_M}}$$

$$N_{\overline{Y}_m} = \sum_q N_{\overline{Y},f_q}$$

$$\nu_{\overline{Y}_m} = \left| \frac{\left(\sum_{q=1}^{N_f} \left[\frac{s^2_{\overline{Y},f_q}}{N_{\overline{Y},f_q}}\right]\right)^2}{\sum_{q=1}^{N_f} \left[\left(\frac{s^2_{\overline{Y},f_q}}{N_{\overline{Y},f_q}}\right)^2 \frac{1}{\nu_{\overline{Y},f_q}}\right]} \right|$$

$$\boxed{s^2_{\overline{Y}_m} = \sum_q s^2_{\overline{Y},f_q}}$$ Sum Variances

Error Factor Analysis

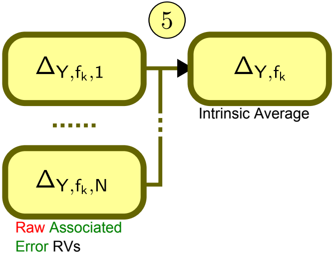
Advantages

- Description of the Error Structure
- Bias Taken Into Account

Drawbacks

- Choice of the Predominant Error Factors

Extra Calculation



Step 5 : Intrinsic average

$\boxed{\overline{\Delta}}$

- Same Error Factor
- Different Y_{true}

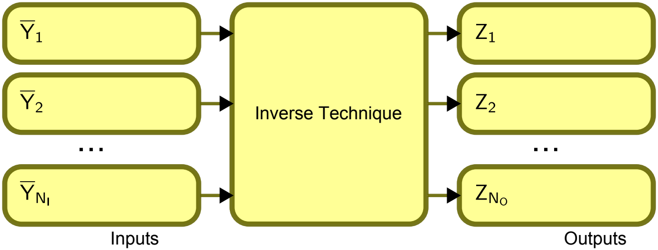
$$N_{Y,f_k} = \sum_i N_{Y,f_k,i}$$

$$\nu_{Y,f_k} = \sum_i \nu_{Y,f_k,i}$$

$$s^2_{Y,f_k} = \frac{\sum_i \nu_{Y,f_k,i} \times s^2_{Y,f_k,i}}{\sum \nu_{Y,f_k,i}}$$

Content of the section

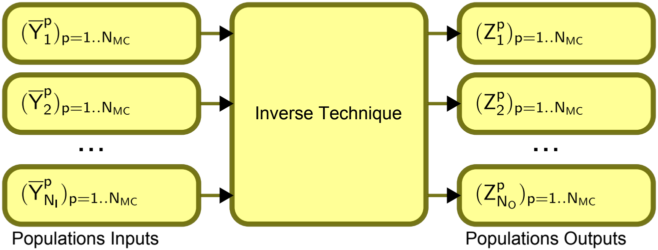
The Monte Carlo Method



Inverse Technique

- N_I Estimation Inputs
- N_O Outputs

The Monte Carlo Method



The Monte Carlo Method

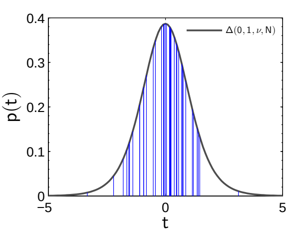
- Noised population for the inputs
- Noised outputs

Generation of Noised Input Populations

$\Delta_{\overline{Y}_m}$ $\xrightarrow{6}$ δ^p $\rightarrow$ $\overline{Y}^p = \overline{Y} + \delta^p$

Estimation Total Error RV — Estimation Total Error Values — Noised Input

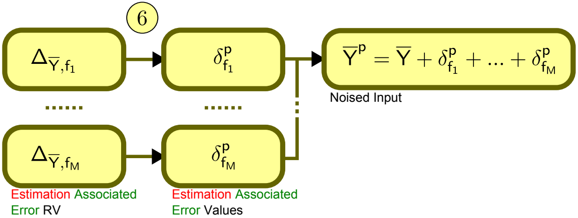
Generation from $\Delta_{\overline{Y}_m}$



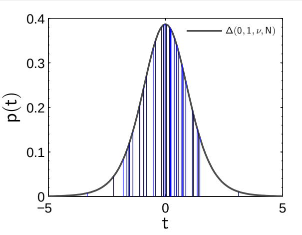
$$\delta^p = t_{rand}^{\nu_{\overline{Y}_m}} \sqrt{s^2_{\overline{Y}_m}}$$

Command `trnd` on MATLAB™

Generation of Noised Input Populations



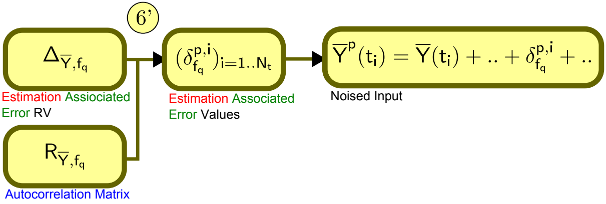
Generation from $\Delta_{\overline{Y}_m}$



$$\delta^p_{f_q} = t_{\mathrm{rand}}^{\nu_{\overline{Y},f_q}} \sqrt{s^2_{\overline{Y},f_q}}$$

Command `trnd` on MATLAB™

Generation of Noised Input Populations

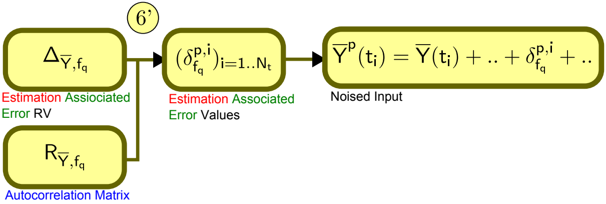


Same estimator $\overline{Y}$, different times t_i

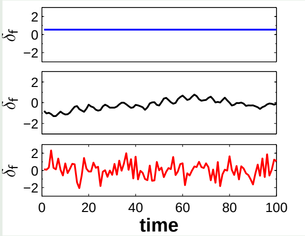
Dependency between :

- $\delta(t_i)$
- $\delta(t_j)$ with $j < i$

Generation of Noised Input Populations



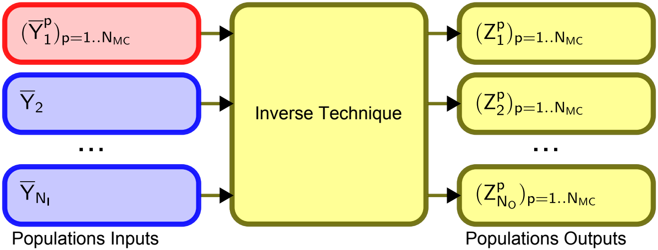
Same estimator $\overline{Y}$, different times t_i



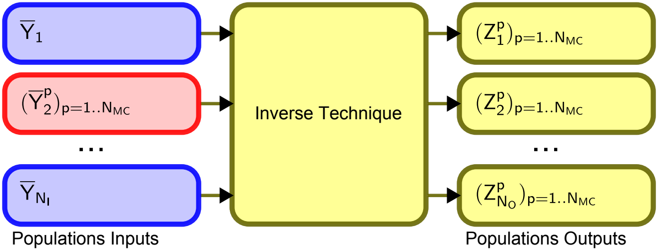
$$(\delta^i) = \mathtt{mvtrnd}(R, \nu)\sqrt{s^2}$$

- Fully dependent : $R_{i,j} = 1$
- Autocorrelated : $R_{i,j} \neq 0$
- Independent : $R_{i,j} = 0$

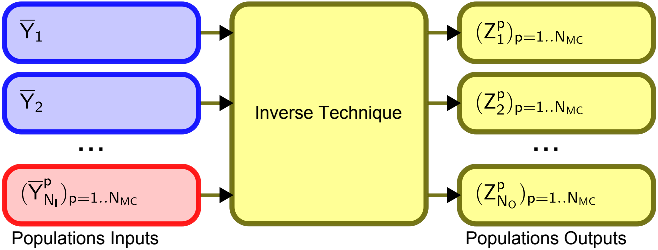
Error Factor Analysis



Error Factor Analysis

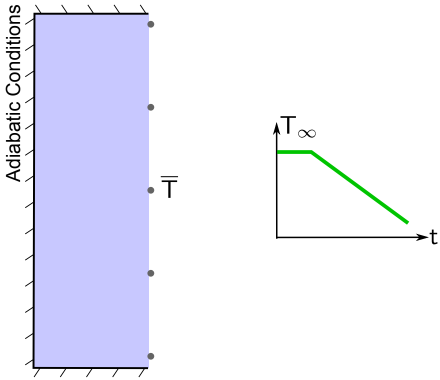


Error Factor Analysis

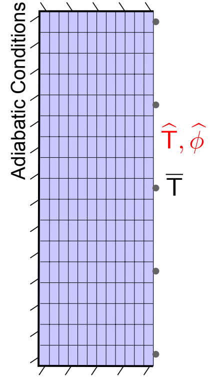


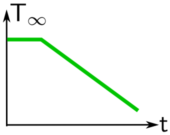
Content of the section

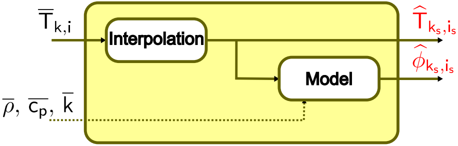
The Inverse Technique



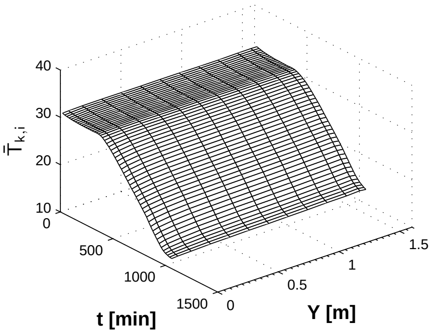
The Inverse Technique



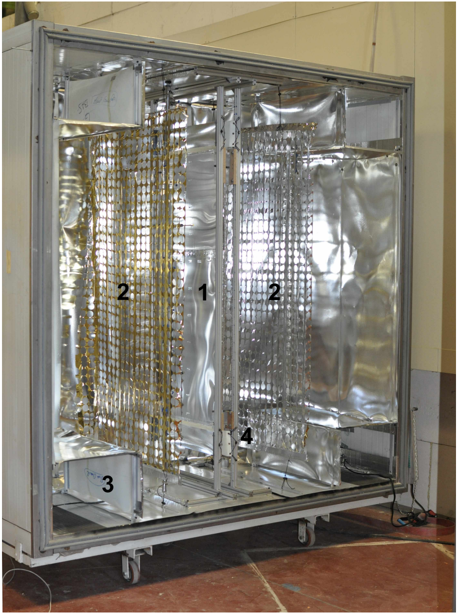




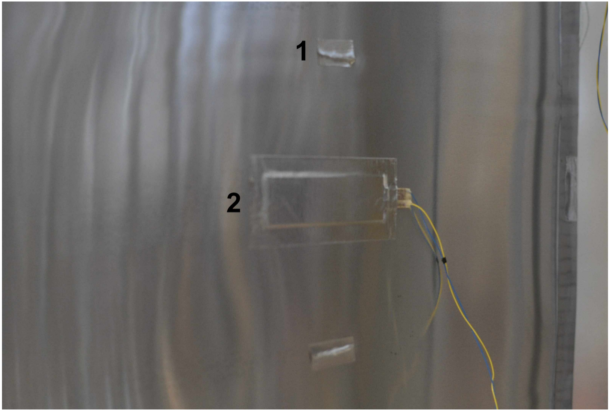
The Inverse Technique



The Inverse Technique

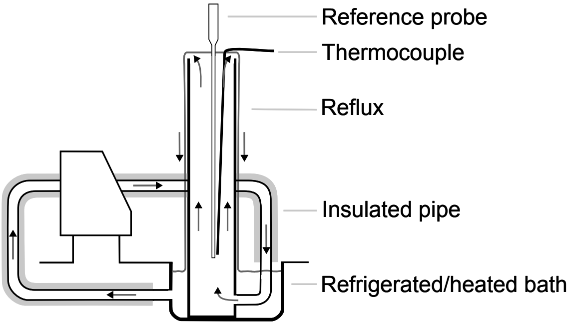


The Inverse Technique



Content of the section

The Calibration of the Thermocouples



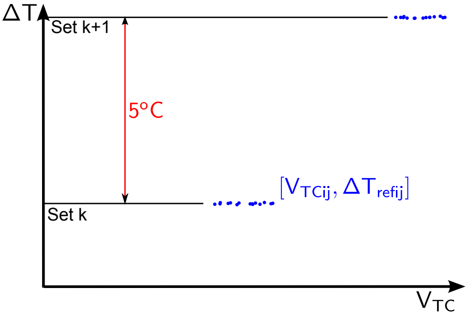
Experimental Setup

The Calibration of the Thermocouples

Data Acquisition

- ΔT : Temperature difference between the two junctions
- V_{TC} : Voltage between the two junctions

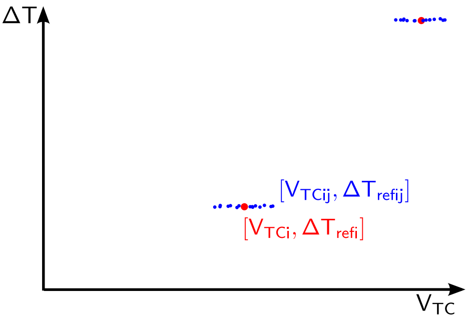
The Calibration of the Thermocouples



Raw data

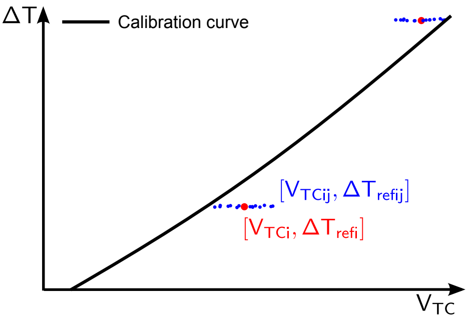
- $N_T = 13$ Sets of acquisition
- $N_A = 15$ Acquisitions per set

The Calibration of the Thermocouples



Averaging the data

The Calibration of the Thermocouples



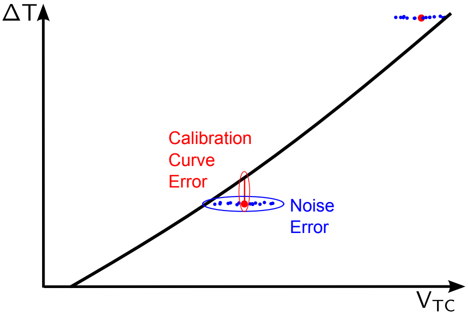
Calibration Curve

- obtained with $[V_{TC,i}, \Delta T_{ref,i}]$

$$\Delta T_c = c_2 V_{TC}^2 + c_1 V_{TC} + c_0$$

Centre de Thermique de Lyon

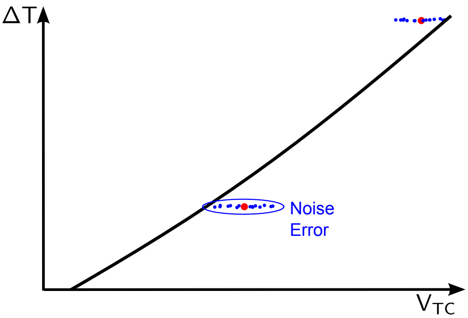
The Calibration of the Thermocouples



The error factors

- Noise Error
- Calibration Curve Error

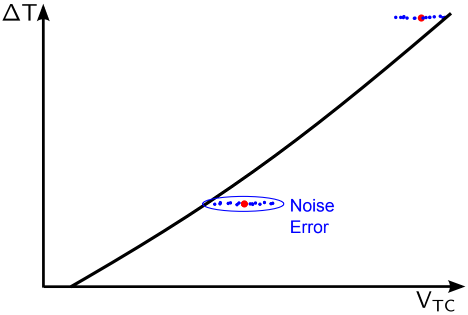
The Noise Error



Description

- Due to the electromagnetic noise
- Fast fluctuating error

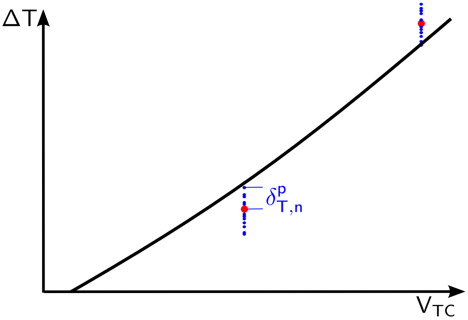
The Noise Error



Approximation of the noise error

$$
\begin{aligned}
V_{TC,i,1} &= V^i_{\text{true}} + \varepsilon^i_{V,c} + \varepsilon^{i,1}_{V,n} \\
V_{TC,i,2} &= V^i_{\text{true}} + \varepsilon^i_{V,c} + \varepsilon^{i,2}_{V,n} \\
& \dots\dots\dots\dots\dots\dots \\
V_{TC,i,N_A} &= V^i_{\text{true}} + \varepsilon^i_{V,c} + \varepsilon^{i,N_A}_{V,n} \\
\hline
\overline{V}_{TC,i} &= V^i_{\text{true}} + \varepsilon^i_{V,c} + \overline{\varepsilon}^i_{V,n}
\end{aligned}
$$

The Noise Error

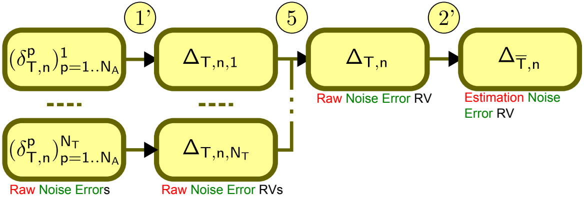


Approximation of the noise error

- Turning a voltage error into a temperature error

$$\delta^{p}_{T,n} = \Delta_c(V_{TC,i,1} - \overline{V}_{TC,i,j}) = \varepsilon^{i,p}_{T,n} - \overline{\varepsilon}^{i}_{T,n}$$

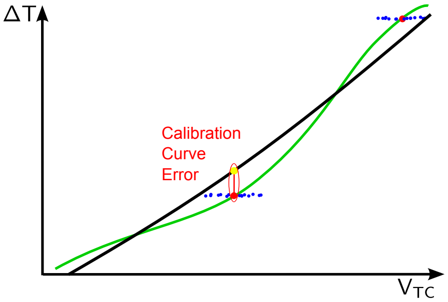
The Noise Error



Calculation steps

- One noise error random variable per set of acquisition
- Step 5 : Intrinsic average
- Step 2' : Fluctuating Error : the Averaging decreases the Variance

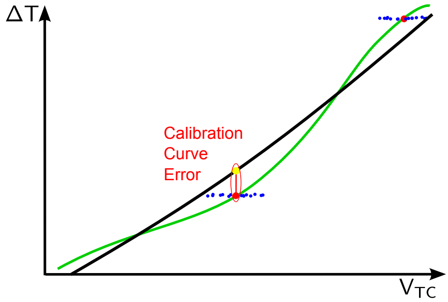
The Calibration Curve Error



Description

- Discrepancy between the curve and the real behavior of the Thermocouple
- Bias during a set of measurement

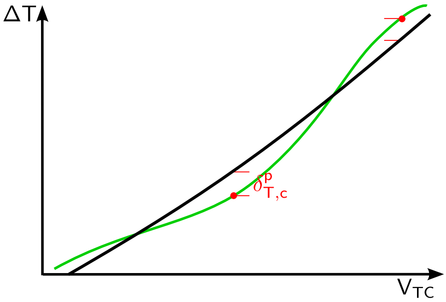
The Calibration Curve Error



Approximation of the calibration curve error

$$\Delta_c(\overline{V}_{TC,1}) = T^1_{\text{true}} + \varepsilon^1_{T,c} + \overline{\varepsilon}^1_{T,n}$$
$$\Delta_c(\overline{V}_{TC,2}) = T^2_{\text{true}} + \varepsilon^2_{T,c} + \overline{\varepsilon}^2_{T,n}$$
$$\cdots\cdots\cdots$$
$$\Delta_c(\overline{V}_{TC,N_T}) = T^{N_T}_{\text{true}} + \varepsilon^{N_T}_{T,c} + \overline{\varepsilon}^{N_T}_{T,n}$$

The Calibration Curve Error

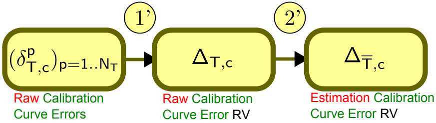


Approximation of the errors

- We suppose $T^{i}_{true} \approx \Delta T_{ref,i}$
- 3 Parameters c_1, c_2, c_3 to estimate the errors :

$$\delta^{p}_{T,c} = \Delta T_c(\overline{V}_{TC,i}) - \Delta T_{ref,i}$$

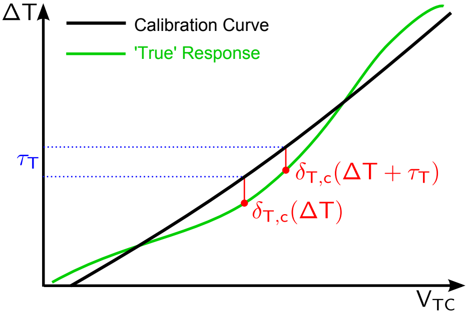
The Calibration Curve Error



Calculation steps

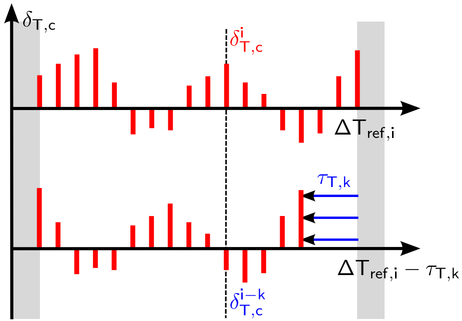
- Step 1' : $\nu_{T,c} = N_T - 3$, because of 3 parameters
- Step 1' : Variance Correction
- Step 2' : Bias : the Averaging does not modify the Variance

The Calibration Curve Error



Autocorrelation

The Calibration Curve Error

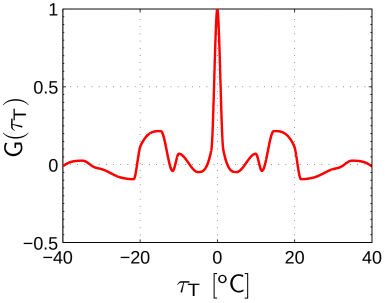


Autocorrelation

- $\tau_{T,k} = k \times 5°\mathrm{C}$ Temperature delay
- Autocorrelation :

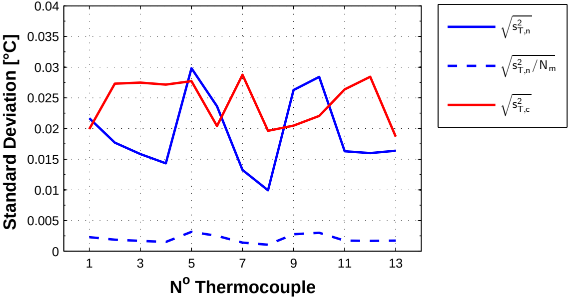
$$G_{T,c}(\tau_{T,k}) = \frac{\sum_{i=1}^{N_A} \delta^{i}_{T,c} \delta^{i-k}_{T,c}}{\sum_{i=1}^{N_A} \left(\delta^{i}_{T,c}\right)^2}$$

The Calibration Curve Error



Autocorrelation

Temperature Estimation Uncertainties

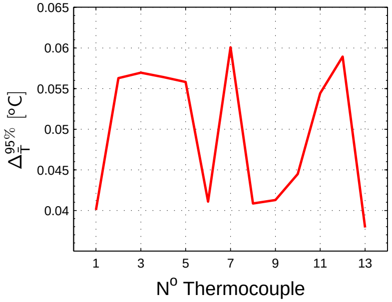


Standard Deviations

- $N_m = 90$

Centre de Thermique de Lyon

Temperature Estimation Uncertainties



Uncertainties

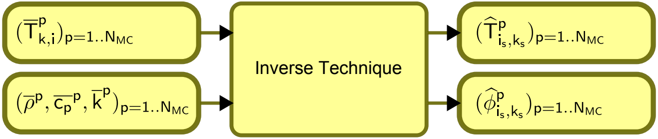
$$T = T_{SF} + \Delta T_c(\overline{V_{TC}})$$

- Error on cold junction neglected
- Average : $\Delta^{95\%}_{\bar{T}_m} = 0.05°\mathrm{C}$

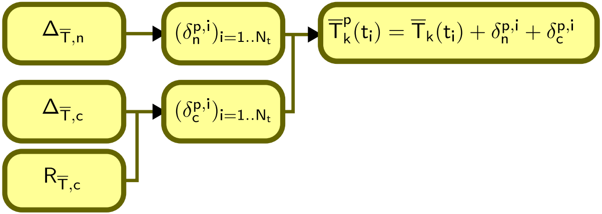
Content of the section

1. Method 0 : Foreword, Definitions
2. Method 1 : Determination of an Estimator Uncertainty
3. Method 2 : The Monte Carlo Method
4. Application 0 : Presentation of Study Case
5. Application 1 : Temperature Estimation Uncertainties
6. Application 2 : Inverse Technique Uncertainties
   - Generation of the Noised Input Populations
   - Temperature Uncertainty
   - Heat Flux Uncertainty

Generation of the Noised Input Populations



Generation of the Noised Input Populations



Noised Temperature

$$\overline{T}_{k,i} = \overline{T}_k(t_i)$$

Generation of the Noised Input Populations

$\Delta_{\overline{T},n}$ → $(\delta_n^{p,i})_{i=1..N_t}$ → $\overline{T}_k^p(t_i) = \overline{T}_k(t_i) + \delta_n^{p,i} + \delta_c^{p,i}$

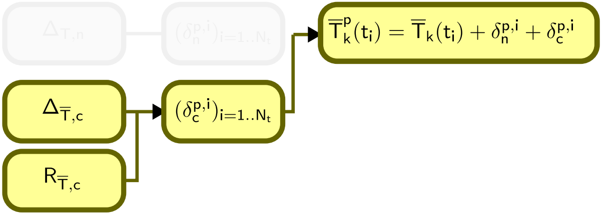
$\Delta_{T,c}$ → $(\delta_c^{p,i})_{i=1..N_t}$

$R_{T,c}$

Noise Errors

- Independent Errors

Generation of the Noised Input Populations

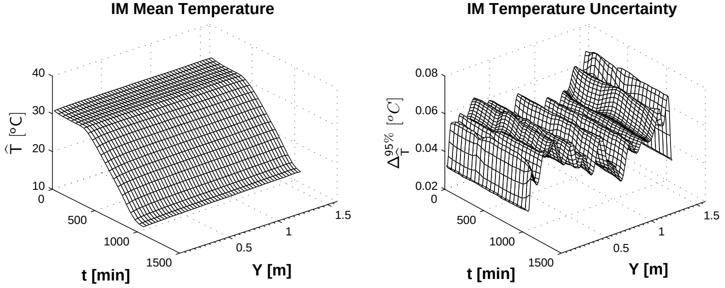


Calibration Curve Errors

- Autocorrelated Errors

$$R_{i,j} = G_T(\overline{T}(t_i) - \overline{T}(t_j))$$

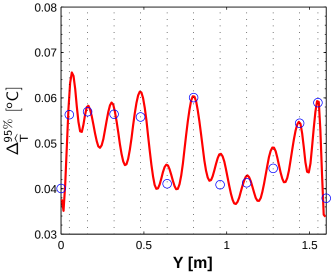
Temperature Uncertainty



Temperature Uncertainty

- $\Delta^{95\%}_{\widehat{T}}$ constant with time

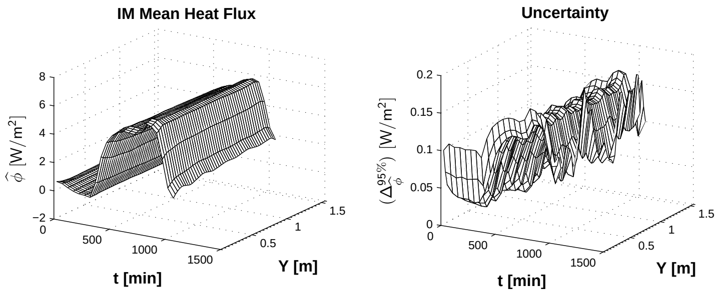
Temperature Uncertainty



Time-Averaged Temperature Uncertainty

- Local Maximums at Thermocouple locations

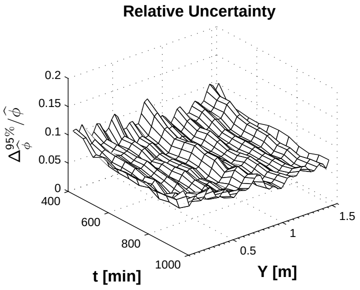
Heat Flux Uncertainty

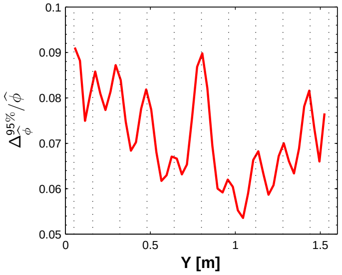


Heat Flux Uncertainty

- $\Delta^{95\%}_{\widehat{\phi}}$ proportional to $\widehat{\phi}$

Heat Flux Uncertainty





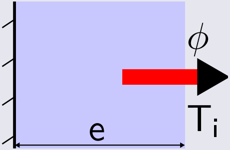
Relative Heat Flux Uncertainty

- $\Delta^{95\%}_{\widehat{\phi}}/\widehat{\phi}$ almost constant with time

Heat Flux Uncertainty : Error Factors

Interpretation

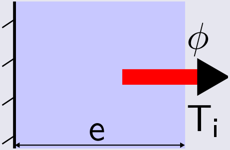
$$\phi \approx e\rho c_p \frac{T_i - T_{i-1}}{\Delta t}$$



Heat Flux Uncertainty : Error Factors

Interpretation

$$\phi \approx e\rho c_p \frac{T_i - T_{i-1}}{\Delta t}$$

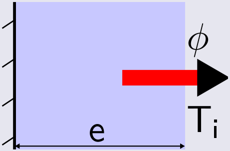


$$\phi + \varepsilon_\phi \approx e(\rho + \varepsilon_\rho)(c_p + \varepsilon_{c_p}) \frac{(T_i + \varepsilon_T^i) - (T_{i-1} + \varepsilon_T^{i-1})}{\Delta t}$$

Heat Flux Uncertainty : Error Factors

Interpretation

$$\phi \approx e\rho c_p \frac{T_i - T_{i-1}}{\Delta t}$$



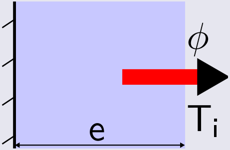
$$\phi + \varepsilon_\phi \approx e(\rho + \varepsilon_\rho)(c_p + \varepsilon_{c_p}) \frac{(T_i + \varepsilon_T^i) - (T_{i-1} + \varepsilon_T^{i-1})}{\Delta t}$$

$$\phi + \varepsilon_\phi \approx \phi + \frac{\varepsilon_\rho}{\rho}\phi + \frac{\varepsilon_{c_p}}{c_p}\phi + (\varepsilon_T^i - \varepsilon_T^{i-1})\frac{e\rho c_p}{\Delta t}$$

Heat Flux Uncertainty : Error Factors

Interpretation

$$\phi \approx e\rho c_p \frac{T_i - T_{i-1}}{\Delta t}$$

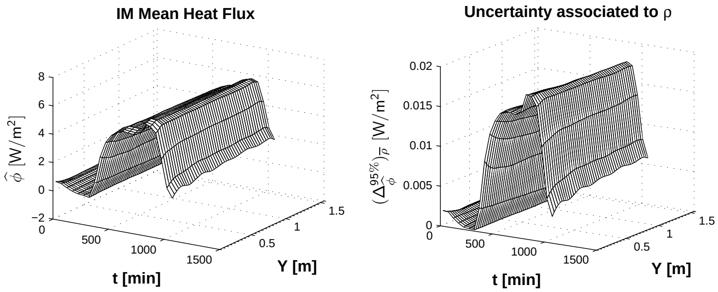


$$\phi + \varepsilon_\phi \approx e(\rho + \varepsilon_\rho)(c_p + \varepsilon_{c_p})\frac{(T_i + \varepsilon_T^i) - (T_{i-1} + \varepsilon_T^{i-1})}{\Delta t}$$

$$\phi + \varepsilon_\phi \approx \phi + \frac{\varepsilon_\rho}{\rho}\phi + \frac{\varepsilon_{c_p}}{c_p}\phi + (\varepsilon_T^i - \varepsilon_T^{i-1})\frac{e\rho c_p}{\Delta t}$$

$$\boxed{\varepsilon_\phi \approx \left[\frac{\varepsilon_\rho}{\rho} + \frac{\varepsilon_{c_p}}{c_p}\right]\phi + (\varepsilon_T^i - \varepsilon_T^{i-1})\frac{e\rho c_p}{\Delta t}}$$

Heat Flux Uncertainty : Error Factors

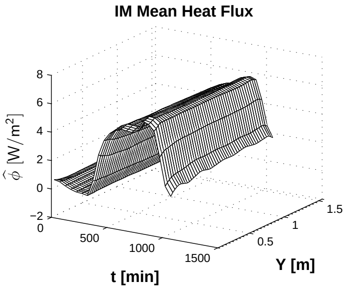


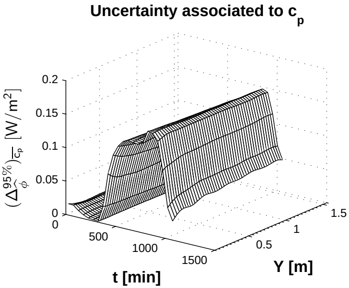
Heat Flux Uncertainty associated to ρ

- proportional to $\widehat{\phi}$

$$\boxed{\varepsilon_\phi \approx \frac{\varepsilon_\rho}{\rho}\phi \propto \phi}$$

Heat Flux Uncertainty : Error Factors



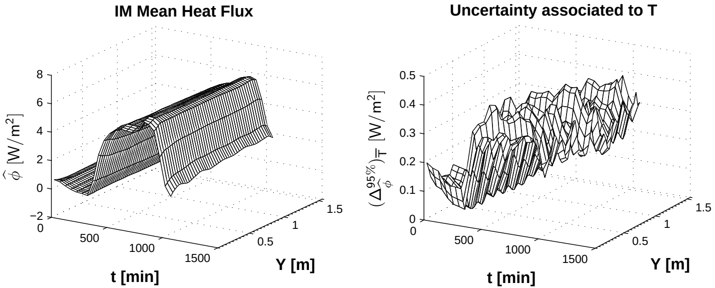


Heat Flux Uncertainty associated to c_p

- proportional to $\widehat{\phi}$

$$\boxed{\varepsilon_{\phi} \approx \frac{\varepsilon_{c_p}}{c_p}\phi \propto \phi}$$

Heat Flux Uncertainty : Error Factors



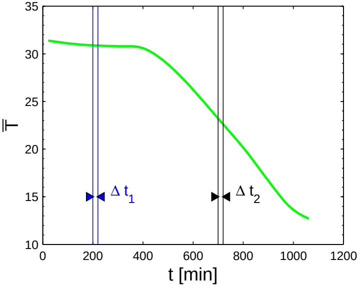
Heat Flux Uncertainty associated to T

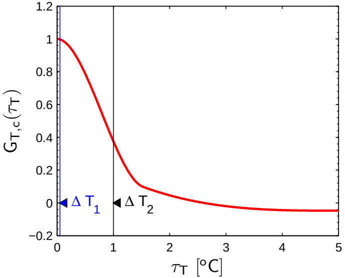
- proportional to $\widehat{\phi}$

$$\boxed{\varepsilon_\phi \approx \left(\varepsilon_T^i - \varepsilon_T^{i-1}\right)\frac{e\rho c_p}{\Delta t} \propto \phi}$$

Centre de Thermique de Lyon

Heat Flux Uncertainty : Error Factors

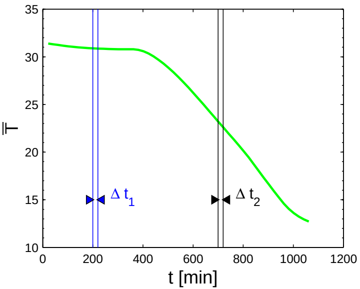


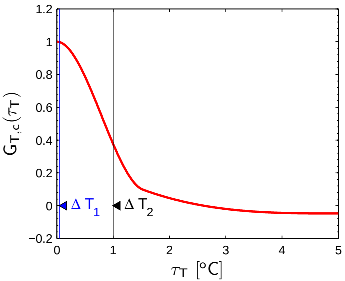


Orders of Magnitude

$$\Delta T_1 < \Delta T_2$$

Heat Flux Uncertainty : Error Factors



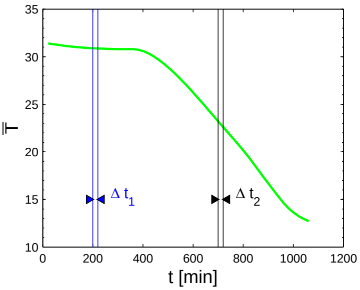


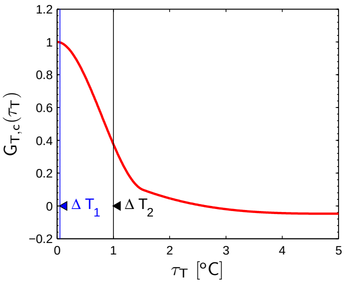
Orders of Magnitude

$$\Delta T_1 < \Delta T_2$$

$$\phi_1 < \phi_2$$

Heat Flux Uncertainty : Error Factors





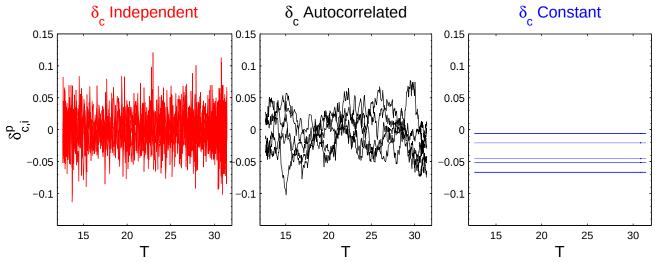
Orders of Magnitude

$$\Delta T_1 < \Delta T_2 \qquad G(\Delta T_1) > G(\Delta T_2)$$

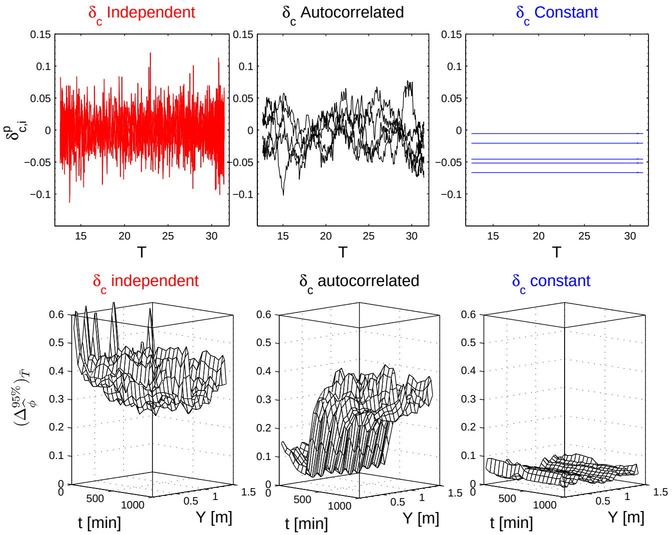
$$\phi_1 < \phi_2 \qquad (\varepsilon_T^i - \varepsilon_T^{i-1})_1 < (\varepsilon_T^i - \varepsilon_T^{i-1})_2$$

$$(\varepsilon_\phi)_1 < (\varepsilon_\phi)_2$$

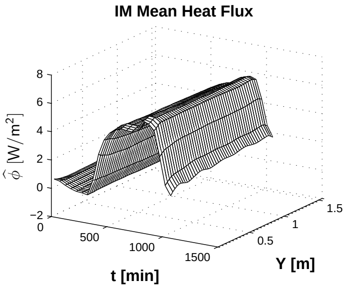
Heat Flux Uncertainty : Error Factors

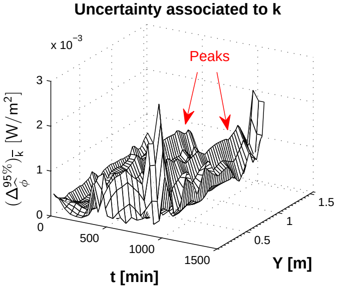


Heat Flux Uncertainty : Error Factors



Heat Flux Uncertainty : Error Factors

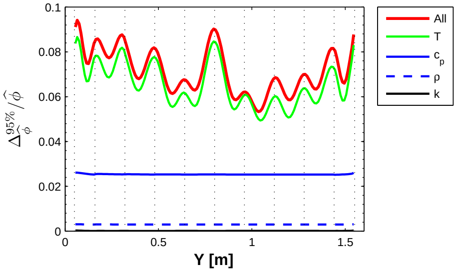




Heat Flux Uncertainty associated to k

- Peaks when $\widehat{\phi}$ varies

Heat Flux Uncertainty : Error Factors



Orders of Magnitude

- Error in k negligible
- Error Mainly due to the Temperature Uncertainties

Centre de Thermique de Lyon

Thanks For Your Attention