Inverse problems in a microchannel

The correlation method

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Research Areas:

- Fluids and Flows
- Transfers and Porous Media
- Energy and Thermal Systems

1. **Objectives of this tutorial**
2. Microfluidics systems
3. Experimental Setup
4. Modeling of a microfluidics system
5. The correlation method
6. Some experimental work
7. Results
8. Conclusion

Main goal : presentation of the correlation method

-Description of the experimental bench

-Microfluidics

-IR Thermography

-Perform real time treatment

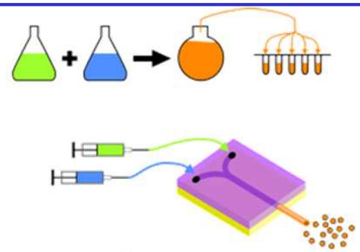
-Matlab

-See the application of the method to different topics

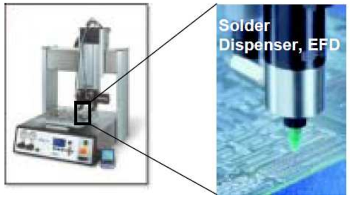
-Flow characterization

-Source term detection (chemical reaction or phase change)

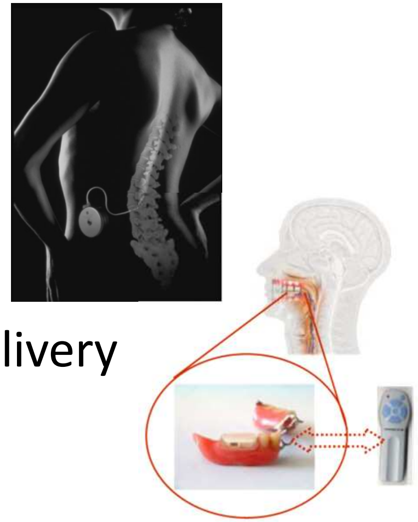
Stakes and challenges



Chemistry (Lab on a chip)

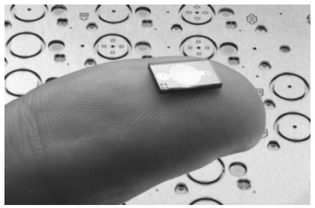


Industrial dispensers



Drug delivery

Microfluidics

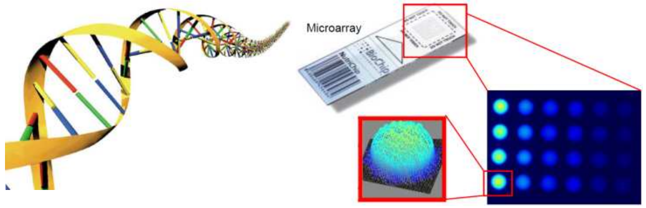


MEMS

(Micro Electro Mechanical Systems)

Everyday life

Biological tests

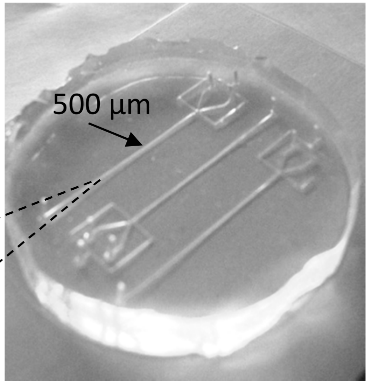


Thermal characterization

Field of application: chemical reaction (Rhodia)

Chemical, pharmaceutical, medical industries…

Challenge → Controlling the reaction
Data acquisition
Safety



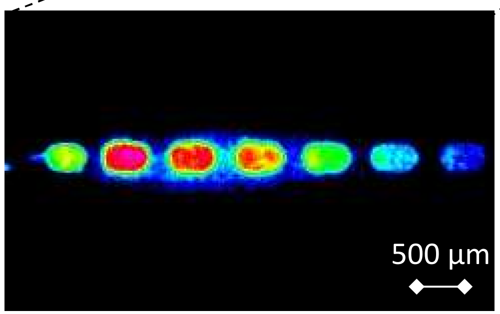
Microfluidics chip

Thermochemical properties in microfluidics

Thermodynamics and

kintetics data

Reactor design



Acid/base droplets reaction, film 600 fps

Thermal analysis tool

Scale (≈ 25 μm)

Quantitative measurements

(≈ μW, mK, ms)

Difficulties :

Instrumentations, inverse methods

Heterogeneous, small sizes, volumes and heat flux…

InfraRed Thermography

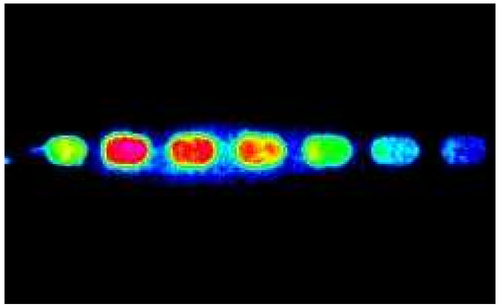
Thermal microscope

Instrumentation for measurement of temperature field

Multiscale

µm-cm

Thermal - Energy



Energy conversion

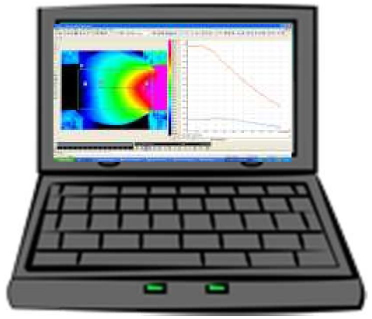
Electronics (µchip), MEMS, Chemistry (µreactors),

Microsystem

Size reduction

Fast acquisition

Data acquisition



Mass data treatment

Inverse methods
Thermal properties
Energy control

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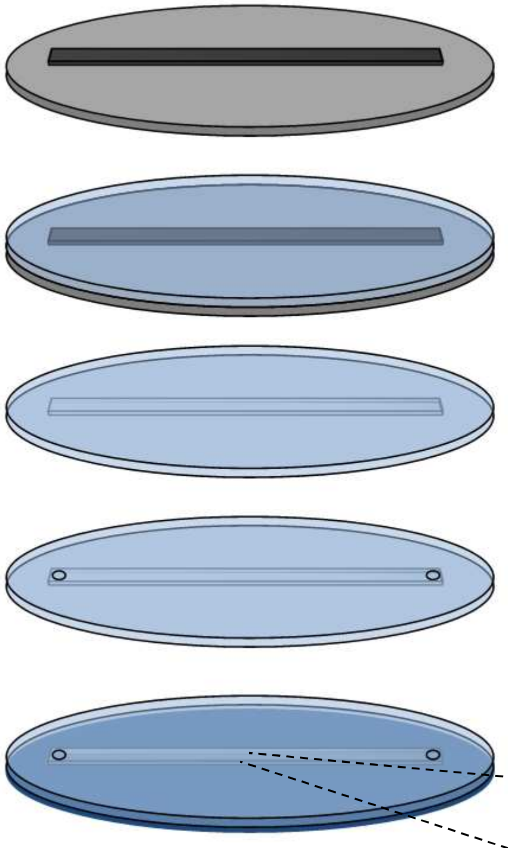
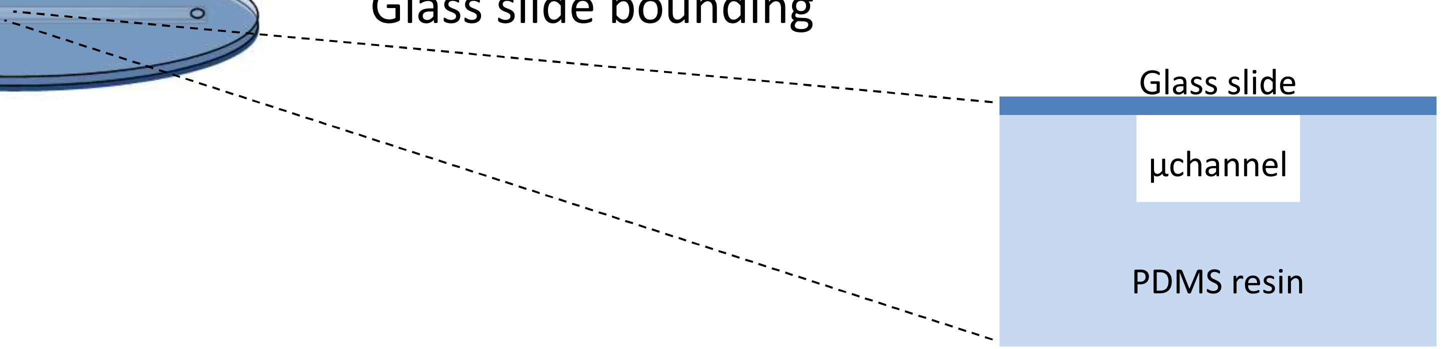
Microfluidics chip

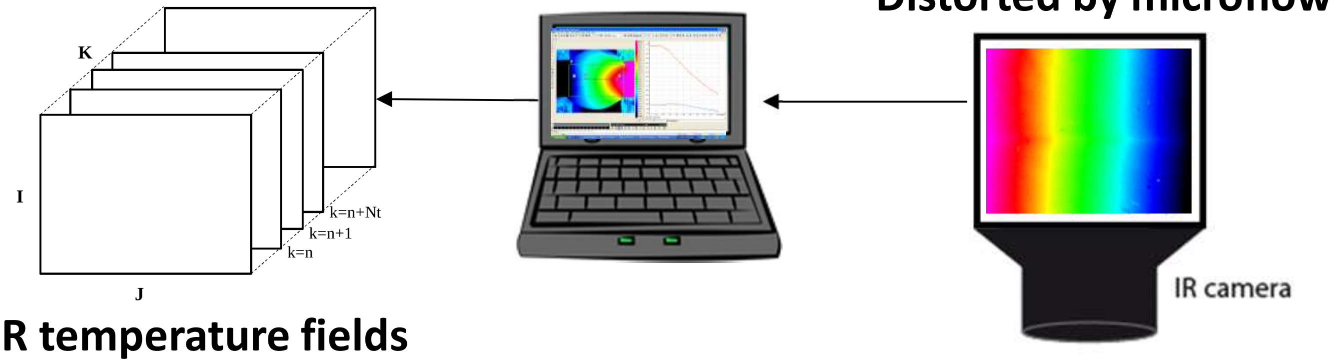


1. Positive mold
2. PDMS resin pouring + reticulation
3. Unmolding
4. Inlet and outlet drilling
5. Glass slide bounding

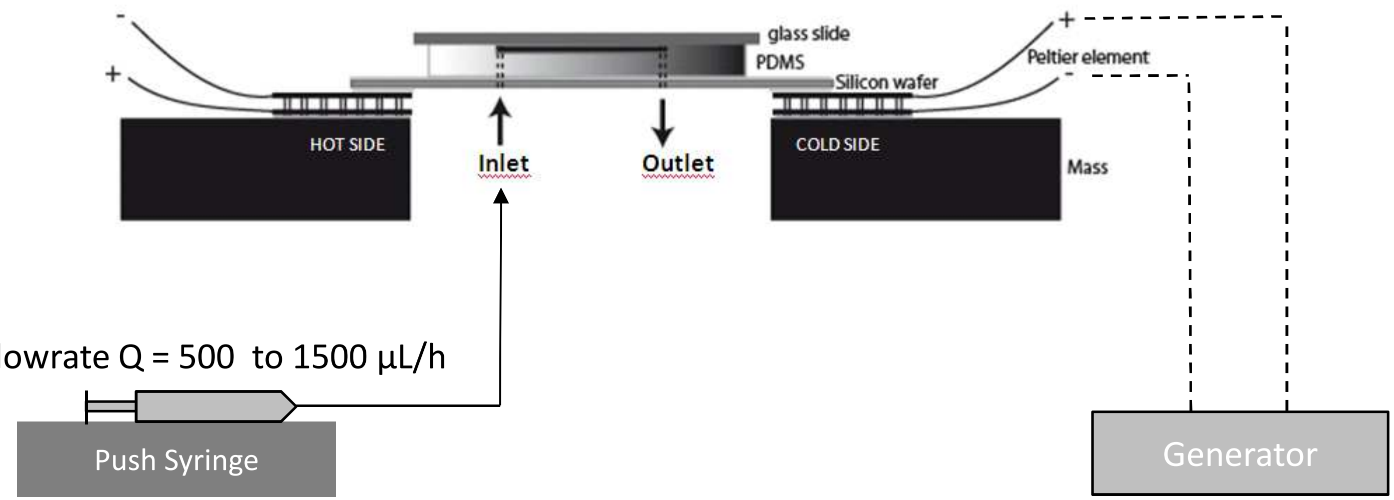


InfraRed Thermography for microfluidics systems

1D thermal gradient
Distorted by microflow



IR temperature fields



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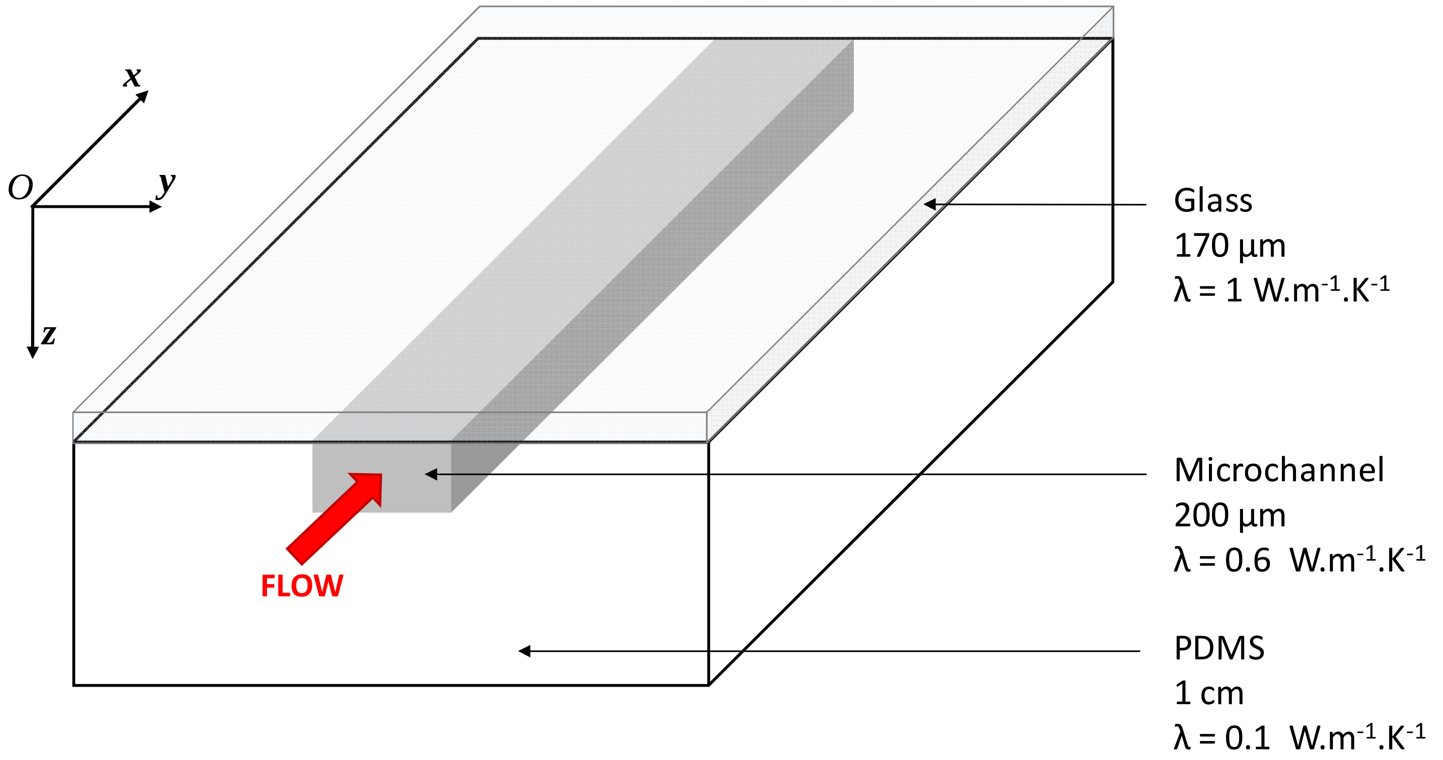
3D scheme of used microfluidics chip



Thin layer
Highly conductive

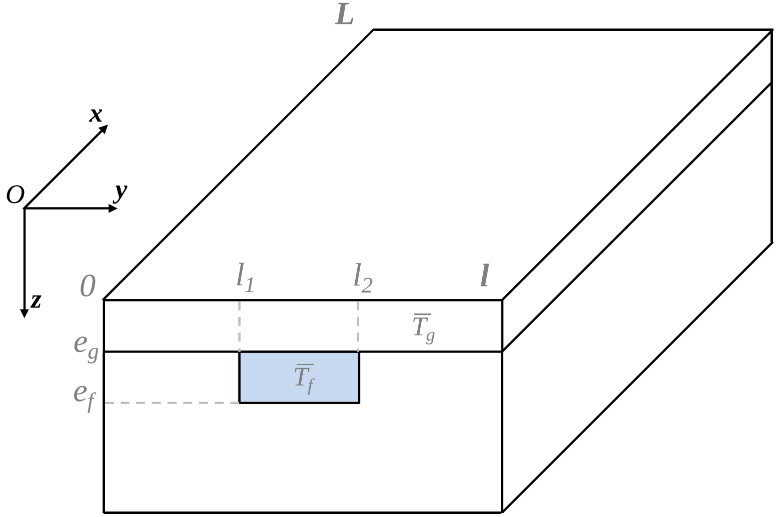
Thick layer
Insulator

Assumptions:

Geometry , thermal properties of layers, flow => 2D model (x,y) for IR measurements

Averaged over z direction

Heat equation inside the microchannel



$$\overline{T}_f(x,y,t) = \int_{e_g}^{e_f} T_f(x,y,z,t)dz$$

$$\left|\lambda_f \frac{\partial T_f(x,y,z,t)}{\partial z}\right|_{e_g}^{e_f} + e_f\overline{\varphi_f}(x,y,t) + \lambda_f e_f\left(\frac{\partial^2 \overline{T}_f(x,y,t)}{\partial x^2} + \frac{\partial^2 \overline{T}_f(x,y,t)}{\partial y^2}\right) = \rho_f Cp_f e_f\left(v(x,y,t)\frac{\partial \overline{T}_f(x,y,t)}{\partial x} + \frac{\partial \overline{T}_f(x,y,t)}{\partial t}\right)$$

Boundary conditions

$$-\lambda_f \frac{\partial \overline{T}_f(x,y,t)}{\partial x} = \phi_C \quad for\ x = 0 \quad and \ -\lambda_f \frac{\partial \overline{T}_f(x,y,t)}{\partial x} = \phi_F \quad for\ x = L$$

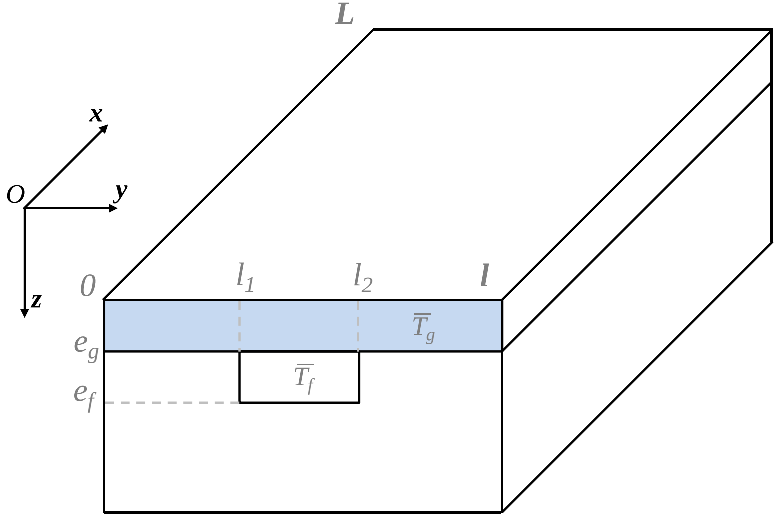
$$\overline{T}_f(x,y,t) = 0 \quad for\ t = 0$$

$$\frac{\partial \overline{T}_f(x,y,t)}{\partial y} = 0 \quad for\ y = l_1 \ \ and\ y = l_2$$

$$-\lambda_f \frac{\partial T_f(x,y,z,t)}{\partial z} = -\lambda_g \frac{\partial T_g(x,y,z,t)}{\partial z} \quad for\ z = e_g$$

$$\frac{\partial T_f(x,y,z,t)}{\partial z} = 0 \quad for\ z = e_f$$

Heat equation inside the glass plate



$$\overline{T_g}(x,y,t) = \int_0^{e_g} T_g(x,y,z,t)dz$$

$$\left|\lambda_g \frac{\partial T_g(x,y,z,t)}{\partial z}\right|_0^{eg} + \lambda_g e_g \left(\frac{\partial^2 \overline{T_g}(x,y,t)}{\partial x^2} + \frac{\partial^2 \overline{T_g}(x,y,t)}{\partial y^2}\right) = \rho_g C p_g e_g \frac{\partial \overline{T_g}(x,y,t)}{\partial t}$$

Boundary conditions

$$-\lambda_g \frac{\partial \overline{T_g}(x,y,t)}{\partial x} = \phi_C \quad for\ x = 0 \quad and \ -\lambda_g \frac{\partial \overline{T_g}(x,y,t)}{\partial x} = \phi_F \quad for\ x = L$$

$$\frac{\partial \overline{T_g}(x,y,t)}{\partial y} = 0 \quad for\ y = 0 \quad and\ y = l$$

$$\overline{T_g}(x,y,t) = 0 \quad for\ t = 0$$

$$-\lambda_f \frac{\partial T_f(x,y,z,t)}{\partial z} = -\lambda_g \frac{\partial T_g(x,y,z,t)}{\partial z} \quad for\ z = e_g$$

$$-\lambda_g \frac{\partial T_g(x,y,z,t)}{\partial z} = h\, T_g(x,y,z,t) \quad for\ z = 0$$

Overall heat equation

Local thermal equilibrium between the averaged temperatures : $\overline{T_g} = \overline{T_f} = T$

$$-h\,T(x,y,0,t) + e_f\overline{\varphi_f}(x,y,t) + (\lambda_f e_f + \lambda_g e_g)\left(\frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial^2 T(x,y,t)}{\partial y^2}\right) = \rho_f Cp_f e_f v(x,y,t)\frac{\partial T(x,y,t)}{\partial x} + (\rho_f Cp_f e_f + \rho_g Cp_g e_g)\frac{\partial T(x,y,t)}{\partial t}$$

Finite differences

$$H_{i,j}^k T_{i,j}^k + \Phi_{i,j}^k + \Delta T_{i,j}^k = Pe^{*k}_{i,j} Tx_{i,j}^k + \left(\frac{1}{Fo^{*k}_{i,j}}\right) Tt_{i,j}^k$$

$$\boxed{\Delta T_{i,j}^k + \Phi_{i,j}^k = Pe^{*k}_{i,j} Tx_{i,j}^k + \left(\frac{1}{Fo^{*k}_{i,j}}\right) Tt_{i,j}^k}$$

$$Pe^{*k}_{i,j} = K1.\frac{v_{i,j}^k \Delta l}{a_{f\,i,j}^k} \quad and \quad K1 = \frac{1}{\left(1 + \frac{\lambda_g e_g}{\lambda_f e_f}\right)} \qquad Fo^{*k}_{i,j} = K2.\frac{a_{f\,i,j}^k \Delta t}{\Delta l^2} \quad and \quad K2 = \frac{\left(1 + \frac{\lambda_g e_g}{\lambda_f e_f}\right)}{\left(1 + \frac{\rho_g Cp_g e_g}{\rho_f Cp_f e_f}\right)}$$

$$H_{i,j}^k = \frac{h_{i,j}^k \Delta l^2}{(\lambda_f e_f + \lambda_g e_g)} \ , \quad \Phi_{i,j}^k = \frac{e_f \overline{\varphi_{i,j}^k} \Delta l^2}{(\lambda_f e_f + \lambda_g e_g)}$$

$$Tx_{i,j}^k = (T_{i+1,j}^k - T_{i,j}^k) \ , \quad Tt_{i,j}^k = (T_{i,j}^{k+1} - T_{i,j}^k)$$

$$\Delta T_{i,j}^k = (T_{i+1,j}^k + T_{i-1,j}^k + T_{i,j+1}^k + T_{i,j-1}^k - 4T_{i,j}^k)$$

Overall heat equation

$$\Delta T_{i,j}^{k} + \cancel{\Phi_{i,j}^{k}} = Pe^{*}{}_{i,j}^{k} Tx_{i,j}^{k} + \cancel{\left(\frac{1}{Fo_{i,j}^{k}}\right) Tt_{i,j}^{k}}$$

What we measure : T

What we want : Φ $\qquad$ Pe $\qquad$ Fo

- Φ → Source term
- Pe → velocity
- Fo → Thermal diffusivity

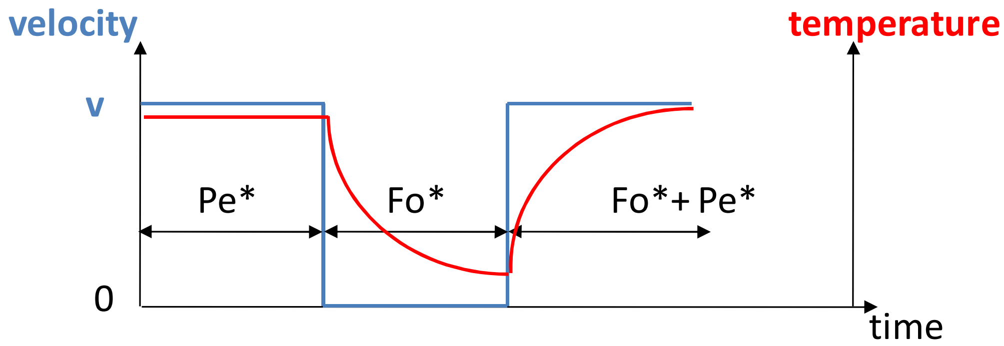
Play with the model

create different operating conditions in order to estimate different parameters.

“Step by step” approach

Without heat source

Steady state	$\Delta T_{i,j}^{k} = Pe{*}_{i,j}^{k} Tx_{i,j}^{k}$	*Calibration of Pe*
Transient state flow OFF	$\Delta T_{i,j}^{k} = \left(\frac{1}{Fo{*}_{i,j}^{k}}\right) Tt_{i,j}^{k}$	*Calibration of Fo*
Transient state flow ON	$\Delta T_{i,j}^{k} = Pe{*}_{i,j}^{k} Tx_{i,j}^{k} + \left(\frac{1}{Fo{*}_{i,j}^{k}}\right) Tt_{i,j}^{k}$	*Calibration of Fo and Pe*



With heat source

Steady state	$\Delta T_{i,j}^{k} + \Phi_{i,j}^{k} = Pe{*}_{i,j}^{k} Tx_{i,j}^{k}$	*Estimation of Pe and Φ*
Transient state	$\Delta T_{i,j}^{k} + \Phi_{i,j}^{k} = Pe{*}_{i,j}^{k} Tx_{i,j}^{k} + \left(\frac{1}{Fo{*}_{i,j}^{k}}\right) Tt_{i,j}^{k}$	*Estimation of Pe, Fo and Φ*

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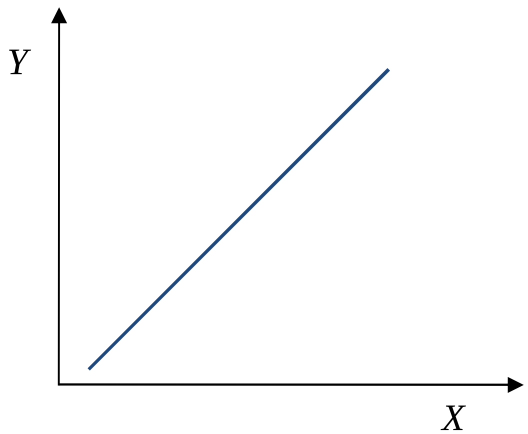
The correlation coefficient: S

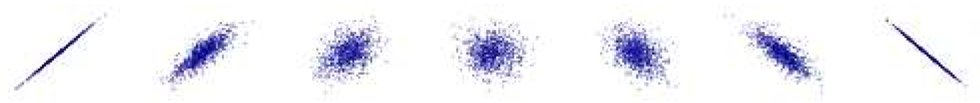
Example with steady state, no heat source:

$$\Delta T_{i,j}^{k} = Pe^{*}{}_{i,j}^{k} Tx_{i,j}^{k} \Longrightarrow Y = A \cdot X$$



Correlation between Y and X: $$S_{Y,X} = \frac{\Sigma(Y-\overline{Y}).(X-\overline{X})}{\sqrt{\Sigma(Y-\overline{Y})^{2}} - \sqrt{\Sigma(X-\overline{X})^{2}}}$$

+1 +0.8 +0.4 0 -0.4 -0.8 -1

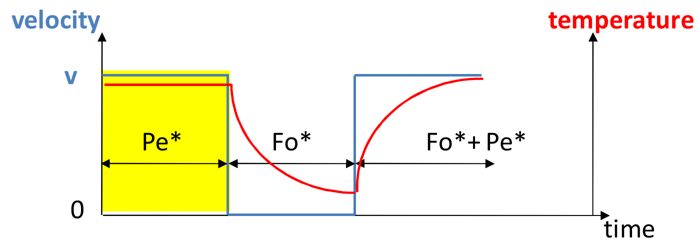


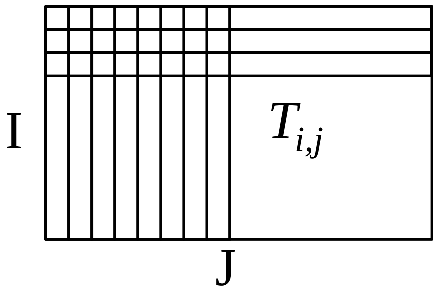
On the same principle, correlation between ΔT and Tx $$Sx_{i,j} = \frac{\sum_{j=n}^{n+Nx} \Delta T_{i,j}.Tx_{i,j}}{\sqrt{\left(\sum_{j=n}^{n+Nx} \Delta T_{i,j}\right)^{2}}.\sqrt{\left(\sum_{j=n}^{n+Nx} Tx_{i,j}\right)^{2}}}$$

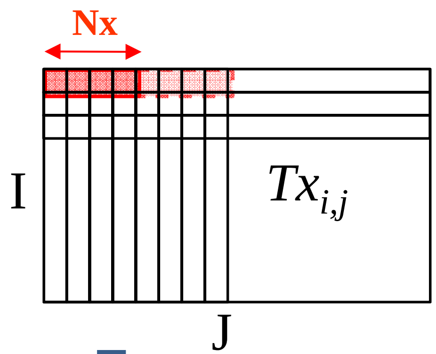
=> Look for values of +1, meaning the model is valid

The spatial correlation coefficient

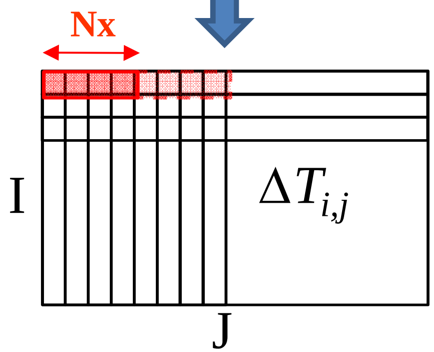
Steady state, no heat source : $\Delta T^{k}_{i,j} = Pe^{*k}_{i,j} Tx^{k}_{i,j}$





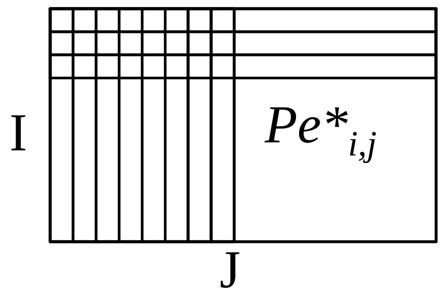


Spatial correlation between ΔT and Tx



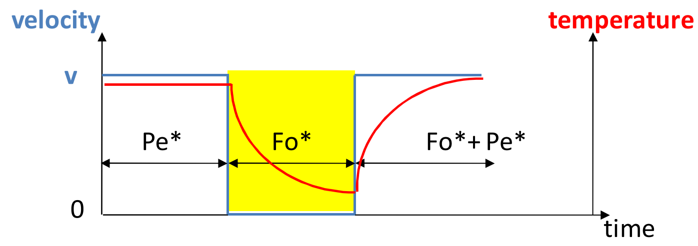
$$Sx_{i,j} = \frac{\sum_{j=n}^{n+Nx} \Delta T_{i,j} . Tx_{i,j}}{\sqrt{\left(\sum_{j=n}^{n+Nx} \Delta T_{i,j}\right)^2} . \sqrt{\left(\sum_{j=n}^{n+Nx} Tx_{i,j}\right)^2}} \quad for \;\; n \in [1; (J - Nx)]$$

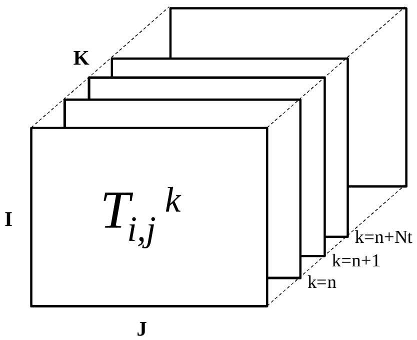
if $Sx_{i,j} = 1$ $\quad Pe^{*}_{i,j} = Sx_{i,j} \dfrac{\sqrt{\left(\sum_{j=n}^{n+Nx} \Delta T_{i,j}\right)^2}}{\sqrt{\left(\sum_{j=n}^{n+Nx} Tx_{i,j}\right)^2}}$

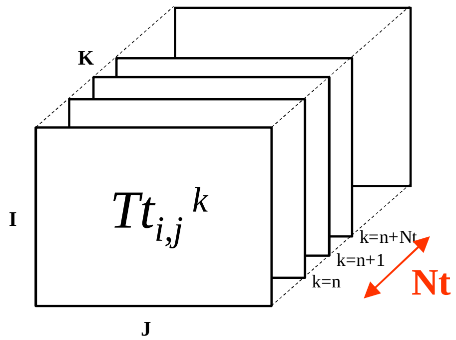


The temporal correlation coefficient

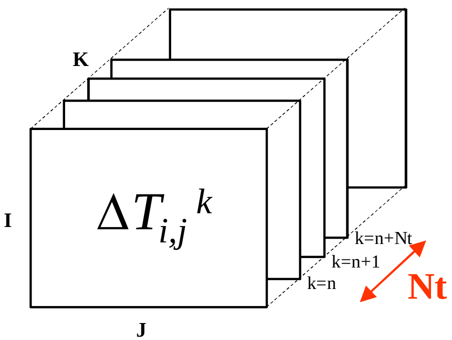
Transient state, no heat source : $\Delta T_{i,j}^k = \left(\frac{1}{Fo^{*k}_{i,j}}\right) Tt_{i,j}^k$





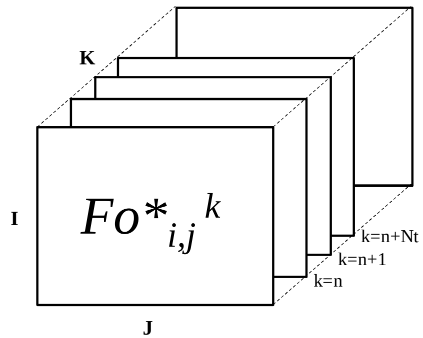


Temporal correlation between ΔT and Tt



$$St_{i,j}^k = \frac{\sum_{k=n}^{n+Nt} \Delta T_{i,j}^k . Tt_{i,j}^k}{\sqrt{\left(\sum_{k=n}^{n+Nt} \Delta T_{i,j}^k\right)^2} . \sqrt{\left(\sum_{k=n}^{n+Nt} Tt_{i,j}^k\right)^2}} \quad for \quad n \in [1; (K - Nt)]$$

if $St_{i,j} = 1$ $\quad \left(\frac{1}{Fo^{*k}_{i,j}}\right) = St_{i,j}^k \frac{\sqrt{\left(\sum_{k=n}^{n+Nt} \Delta T_{i,j}^k\right)^2}}{\sqrt{\left(\sum_{k=n}^{n+Nt} Tt_{i,j}^k\right)^2}}$



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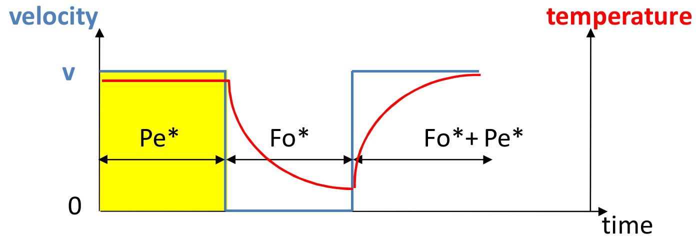
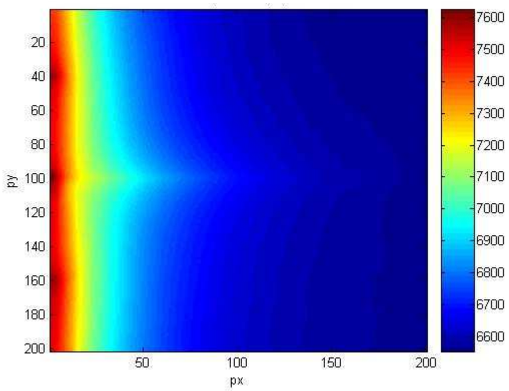
Experiment 1 : Steady state



T

Q = 500 μL/h



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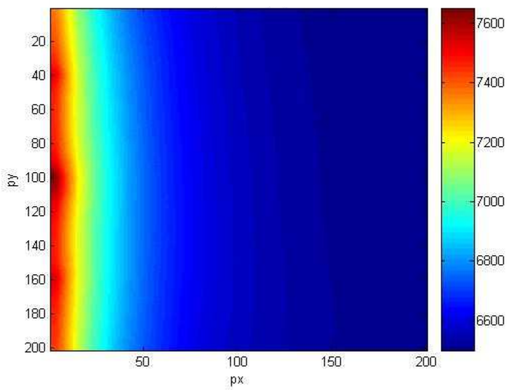
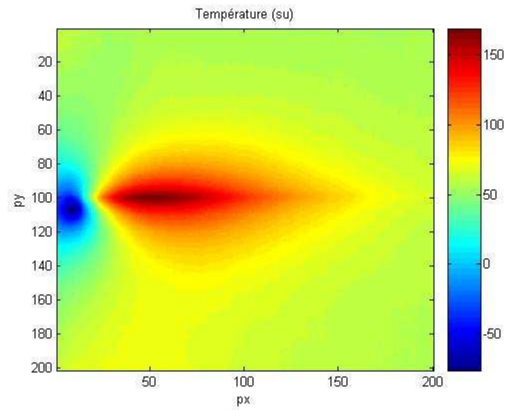
T0

Q = 0 μL/h

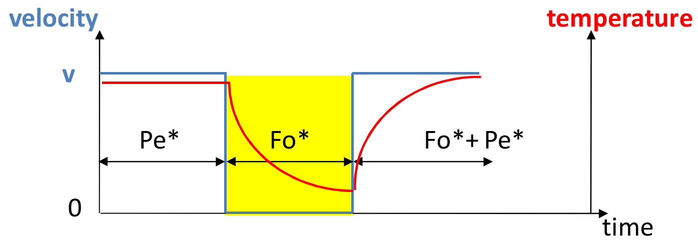


=

(T-T0)

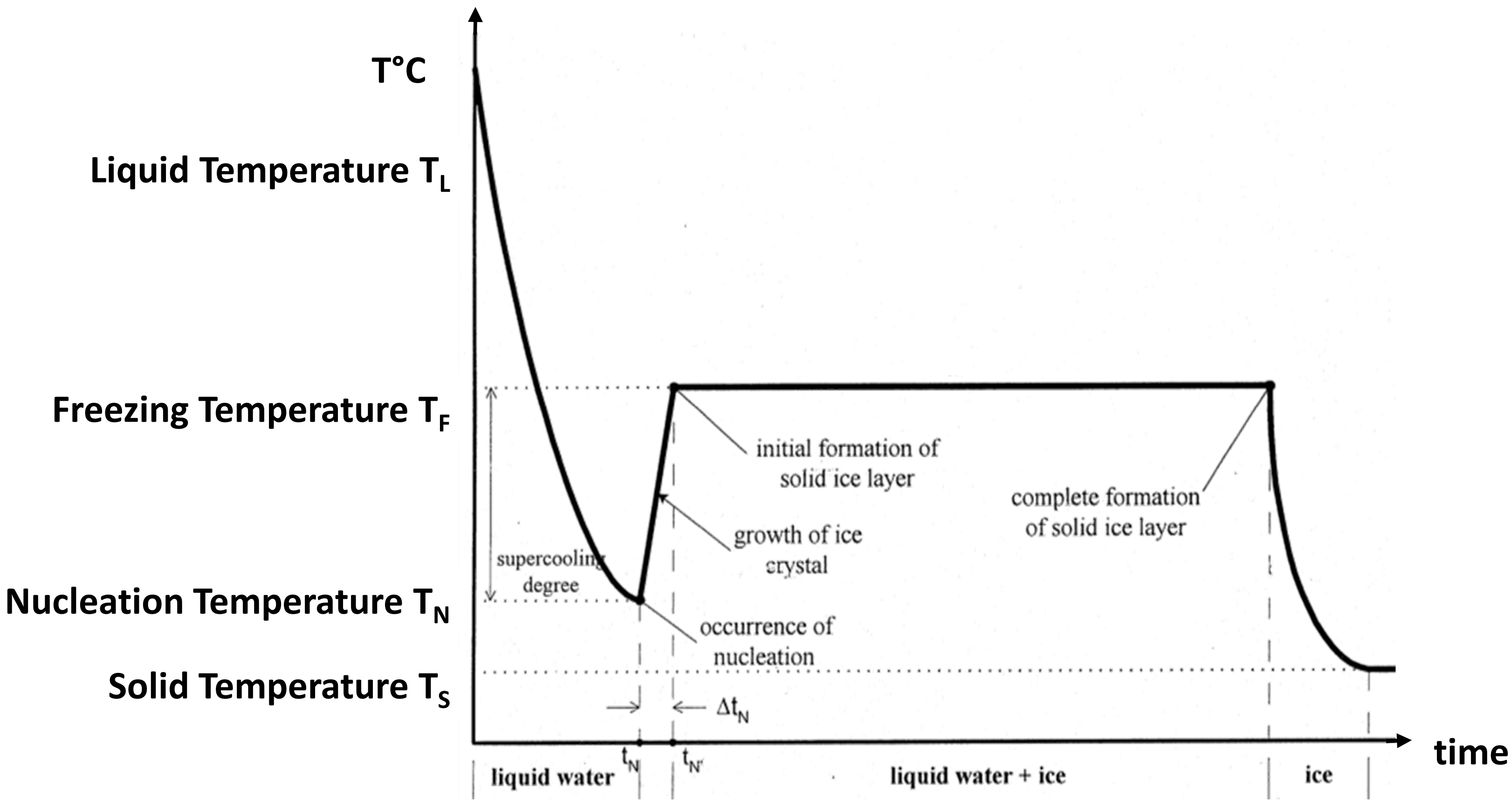


Experiment 2 : Transient state



Experiment 3 : Phase change

The supercooling theory:



MATLAB, temperature fields processing

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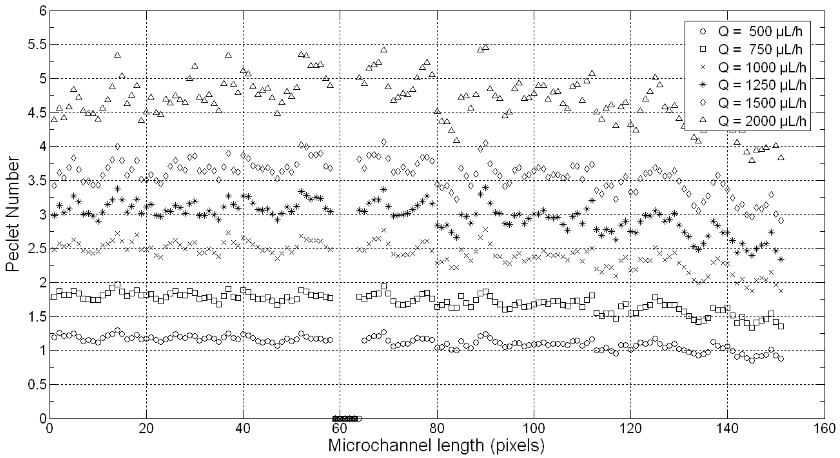
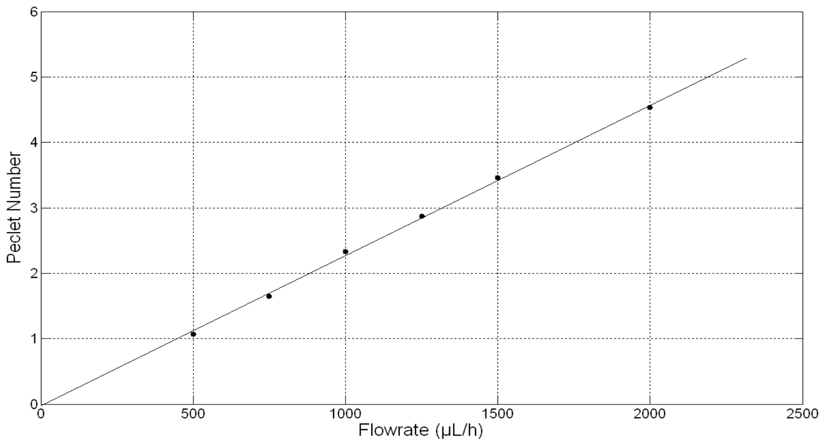


-Estimated Peclet number quite stable along the microchannel.

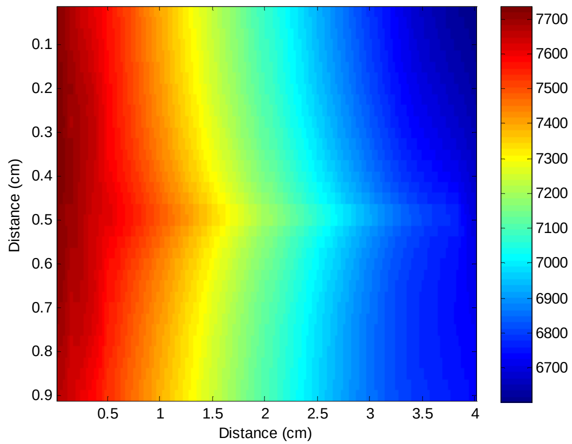
-Higher flow rates are not stable : deformation of PDMS walls



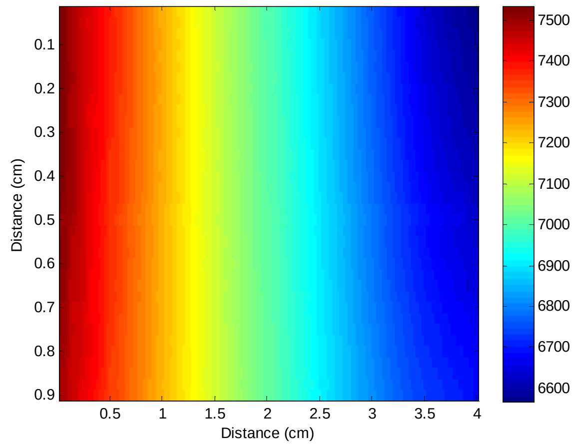
-Linear evolution consistent with physics

Experiment 2: Transient state

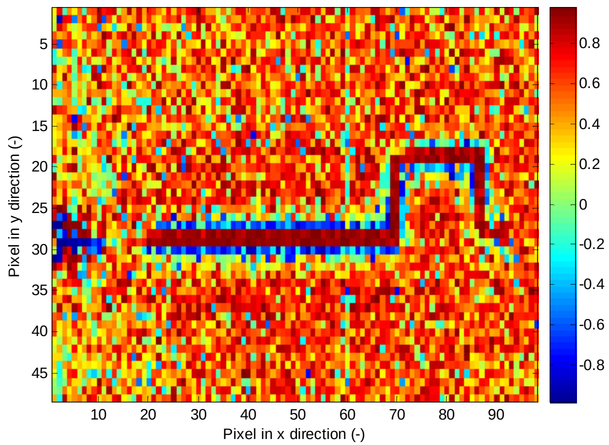
Initial frame



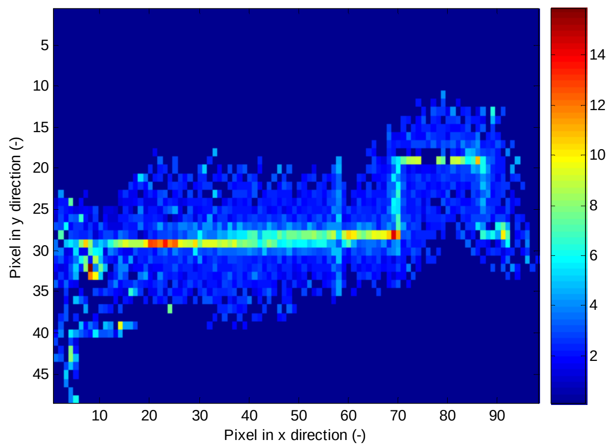
Final frame



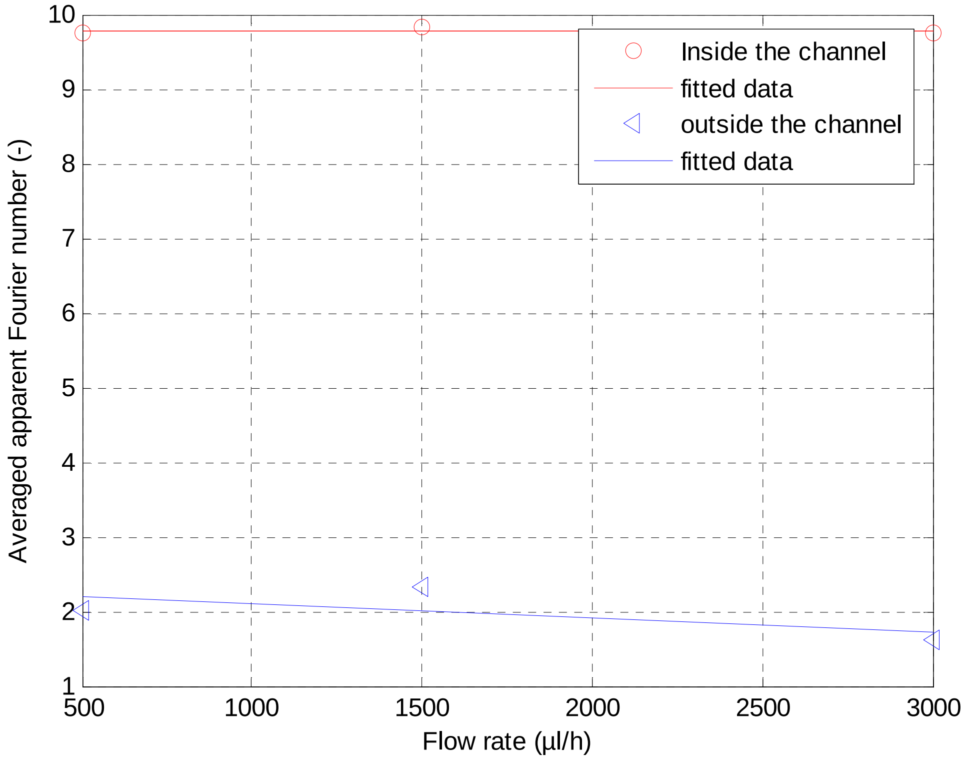
Instant correlation coefficient



Averaged estimated Fo mapping



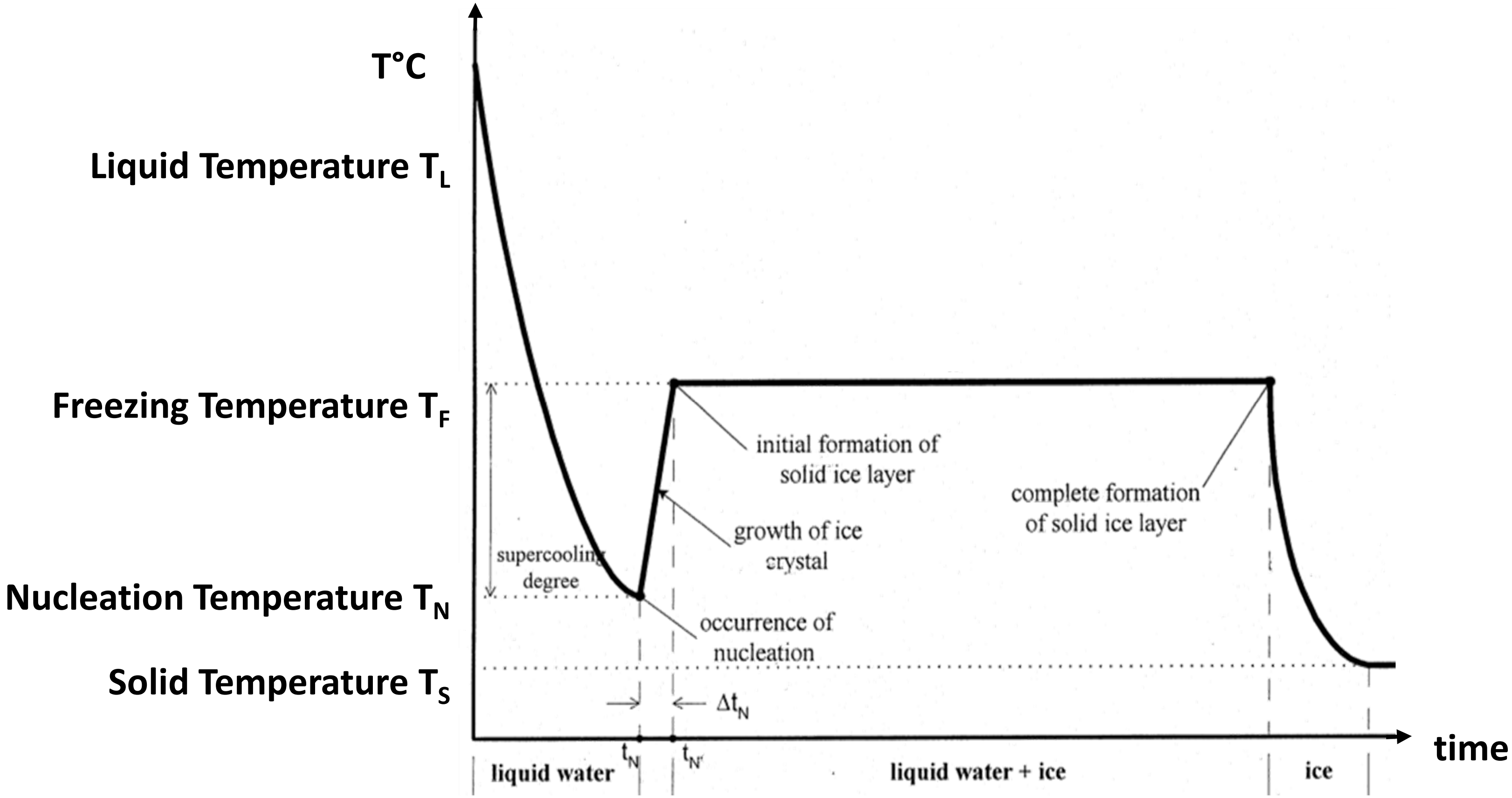
Experiment 2: Transient state



-Estimated Fo number not affected by the flow rate.

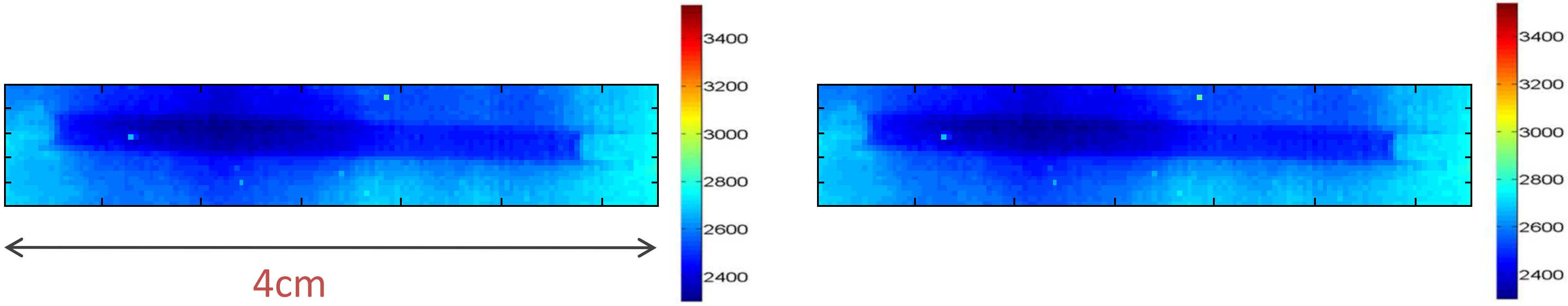
-Difference between inside (water+glass) and outside (PDMS+glass) the microchannel.

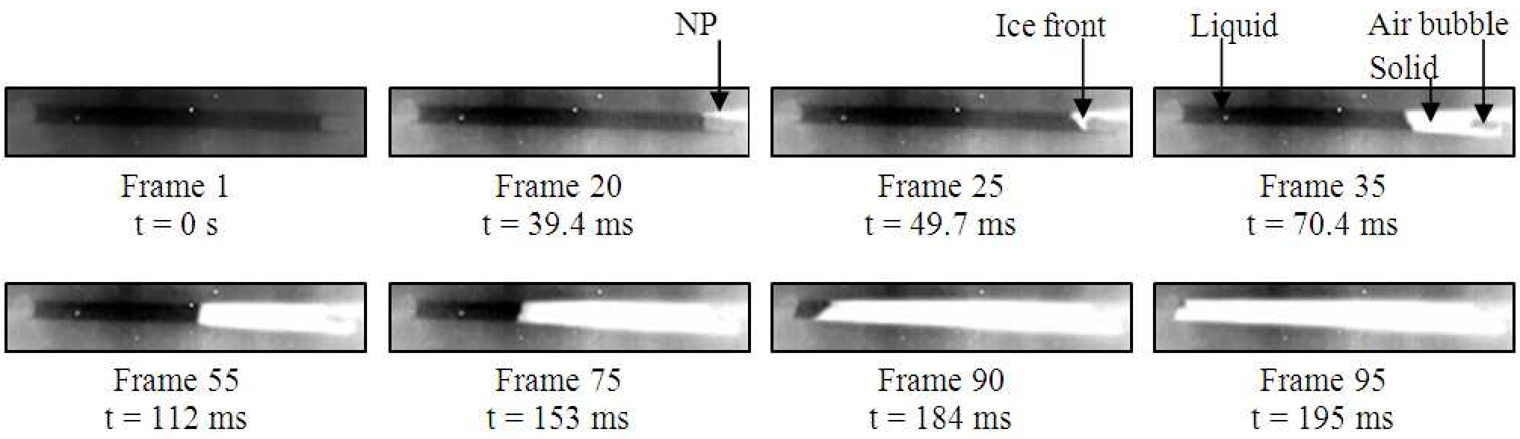
Experiment 3: Phase change



Experiment 3: Phase change

High frequency recordings : 484 Hz

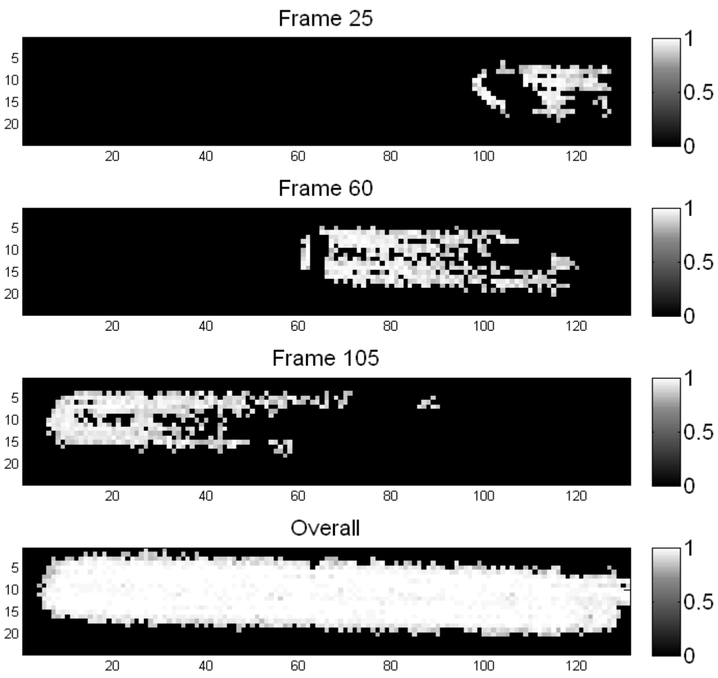




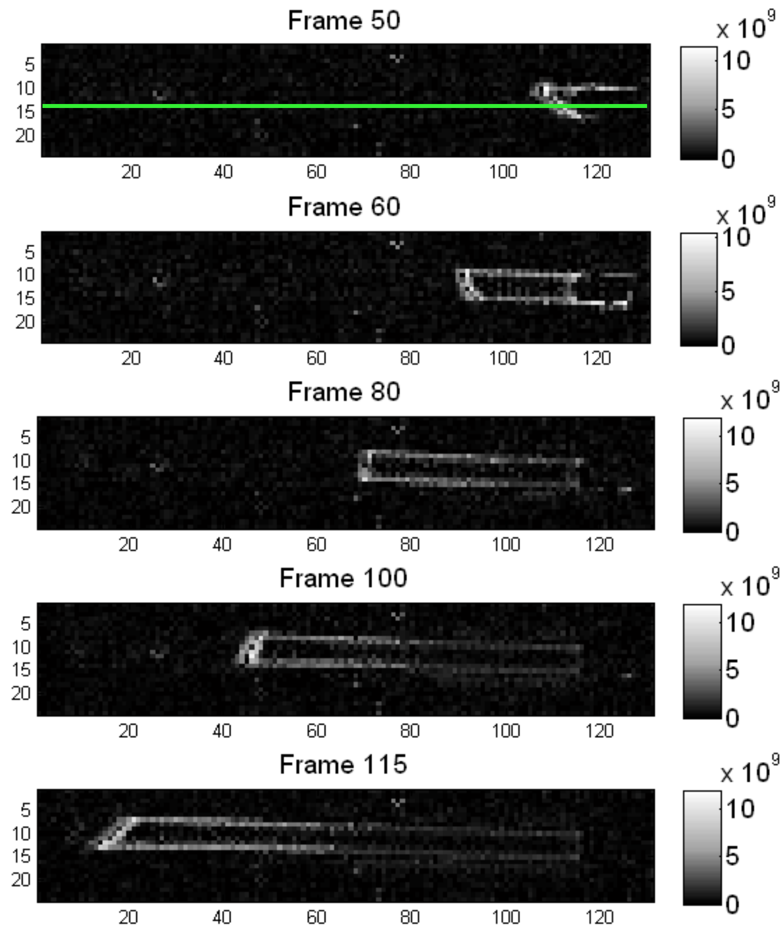
Question: How can we localize the heat source ?

Experiment 3: Phase change

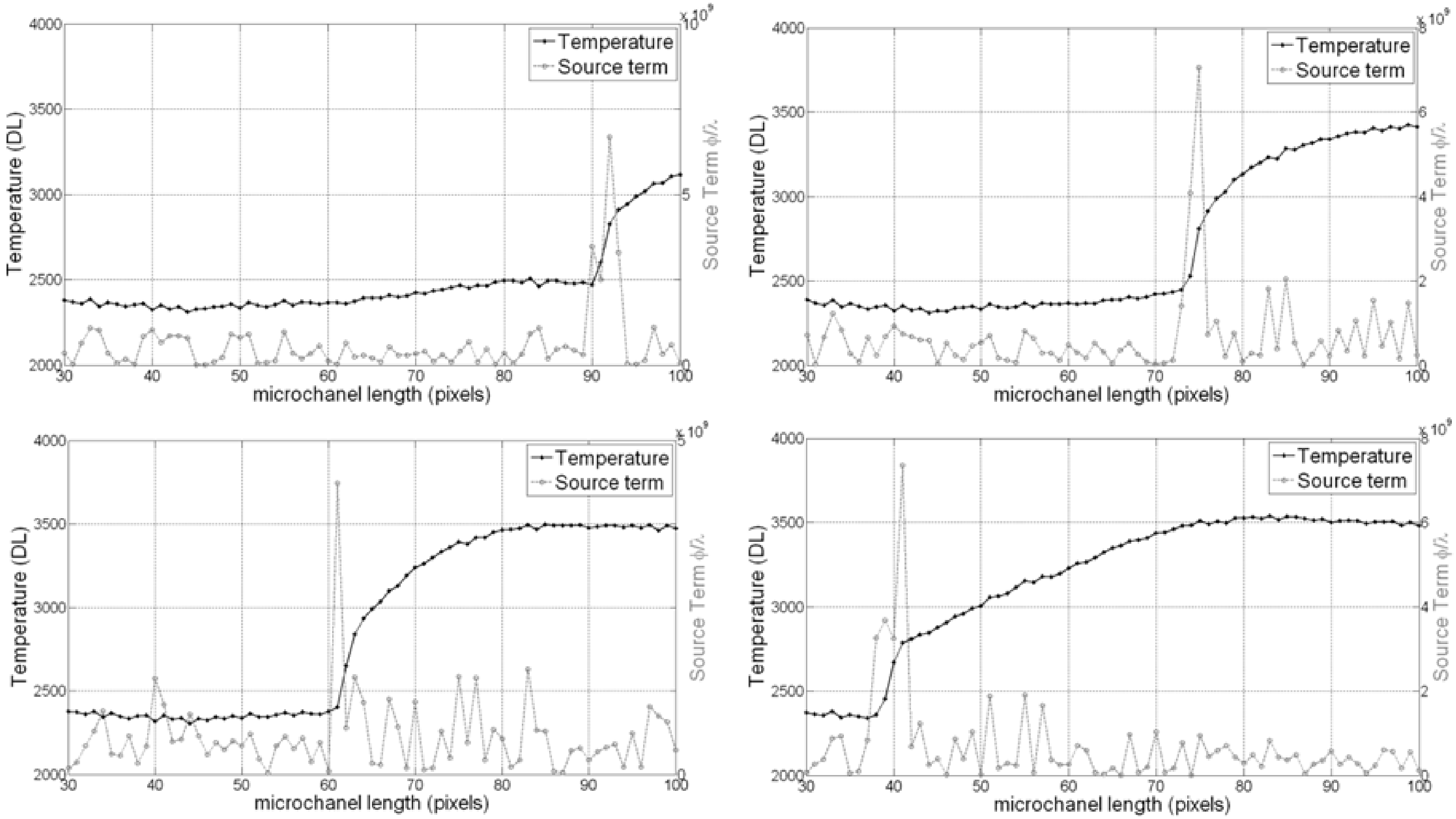
Correlation coefficient



Estimation of source term



Experiment 3: Phase change



The correlation method

Advantages :

Simple and fast processing

No calibration of temperature is needed (temperature variations)

IR Thermography => mapping of estimated parameters

Large field of applications (chemistry, crystallization, blood flows…)

Drawbacks:

Qualitative method (for the moment)

Perspectives:

Reduction of noise and filtering of signal (spatial and temporal)

SVD + convolution and/or lowpass

Quantitave estimations

Improvement of derivations

Inverse problems in a microchannel

The correlation method

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Research Areas:
- Fluids and Flows
- Transfers and Porous Media
- Energy and Thermal Systems