LTIE
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T5: Characterization of transient distributed surface sources through infrared thermography

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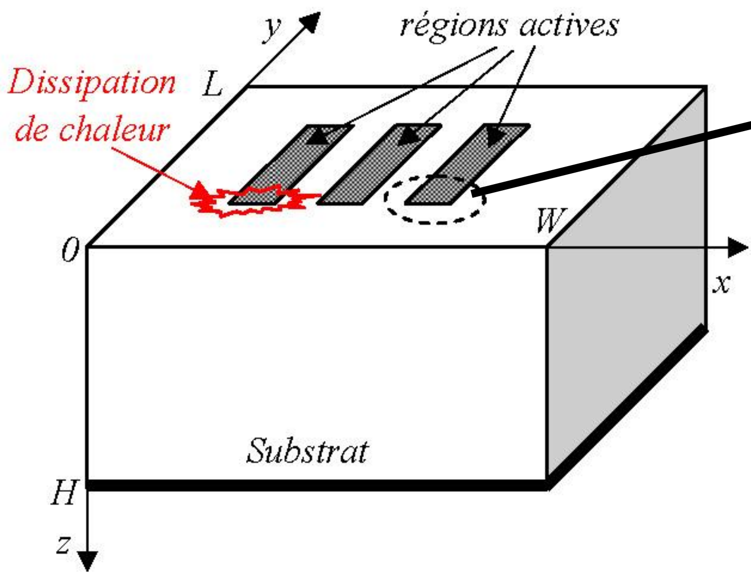
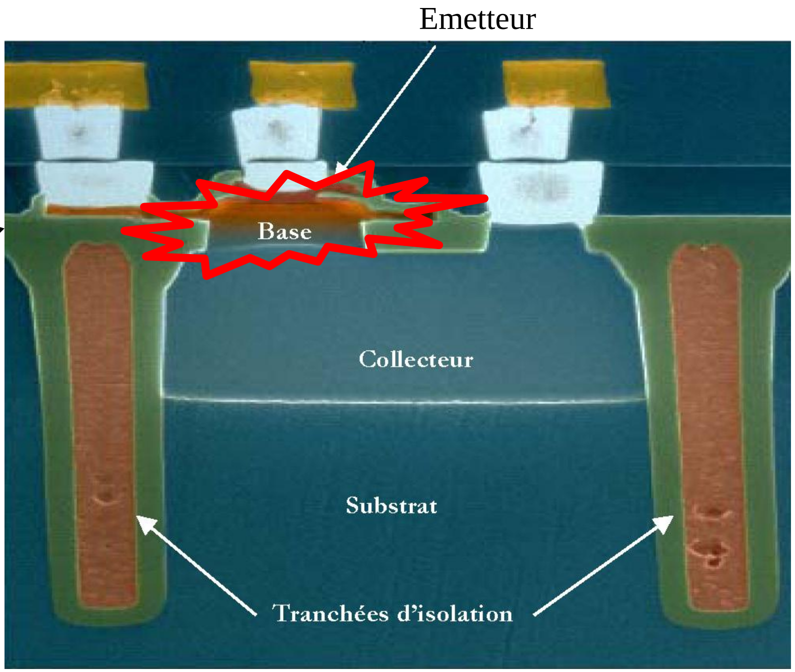
Metti 5, Roscoff

Thermal Phenomenon in electronic components:

Transistors



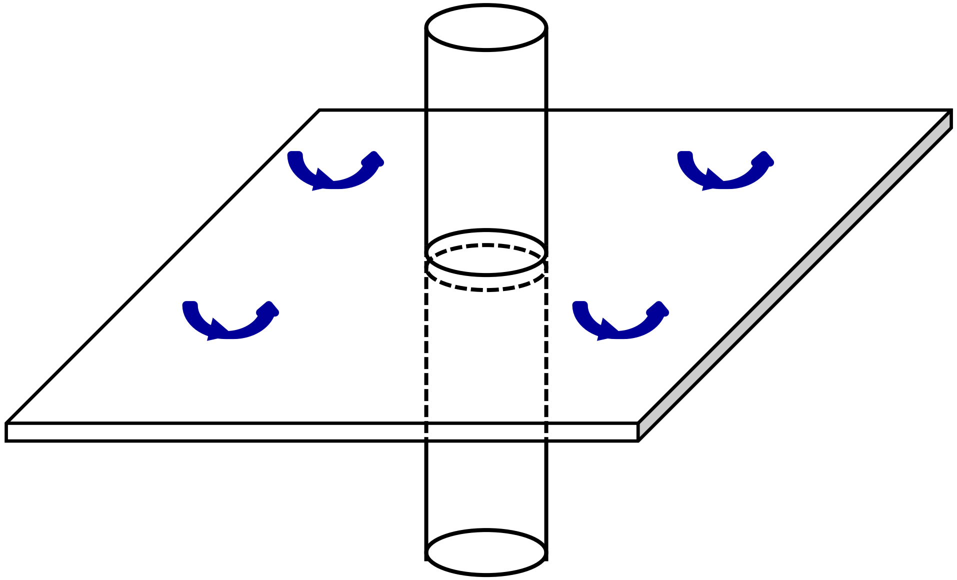


Zoom on active region: real cutting of a BiCMOS SiGe structure 0.25x0.65[μm^2]
(base thickness 0.1[μm]), ft max : 90Ghz.

High frequency signal amplification (500[Ghz] [IEF])

Heat exchangers : ex. finned tubes

Solid / fluid



Heat flux measurements: → **performance**

Characterization of transient distributed surface sources through infrared thermography

I. Calculation tools

- **I.1. Mathematical Modelling**
- **I.2. Spatiale discretization, state representation**
- **I.3. IHCP Description, a way to solve it**
- **I.4. Stabilization / regularization techniques**

II. Numerical validation with simulated data

III. Experimental bench

Characterization of transient distributed surface sources through infrared thermography

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I.3. IHCP Description, a way to solve it

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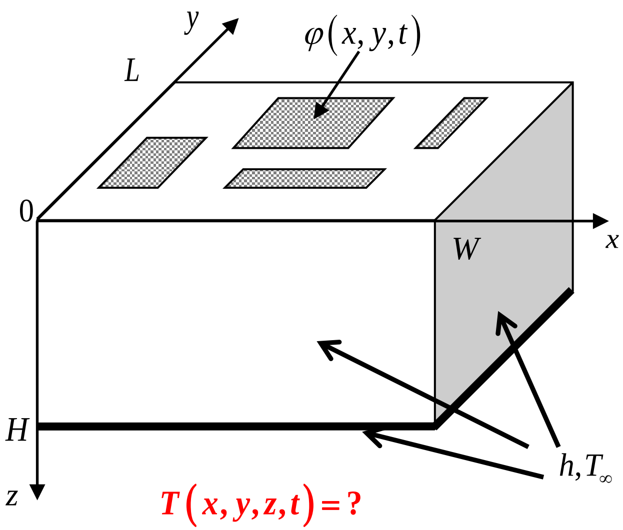
I.1. Mathematical modelling



DHCP: we seek $T(x,y,z,t)$

$$\text{PDE :}\quad \nabla k \underline{\nabla} T - \rho c_p \frac{\partial T}{\partial t} = 0$$

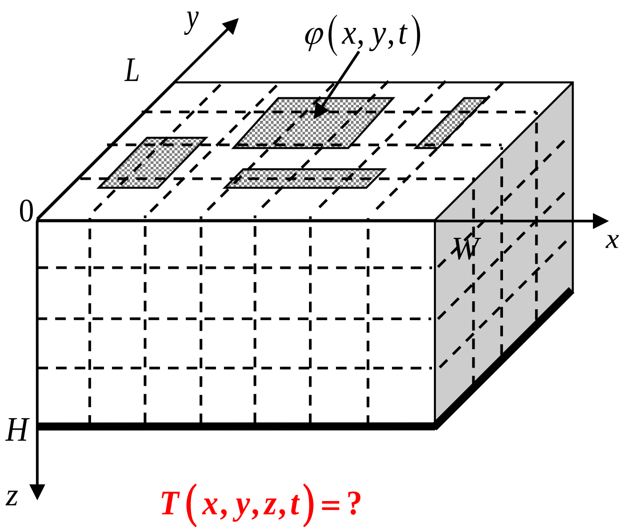
$$\text{IC :}\quad T\big|_{t=0} = F_0(x,y,z)$$

$$\text{BCs :}\quad k\underline{\nabla}T.\underline{n} + hT + f = 0$$

FV Discretization in 3D and transient state

Matrix Eq. dim *Nm* (mesh size) $\boxed{\underline{\underline{A}}\,\underline{T} + \underline{\underline{B}}\,\underline{U} + \underline{\underline{K}}\,\underline{\dot{T}} = 0}$ + CI

I.2. Spatial Discretization, state representation



DHCP: we seek *T(x,y,z,t)*

Integration : PDE : $$\int_{\Omega}\left[\nabla k\underline{\nabla}T-\rho c_p\frac{\partial T}{\partial t}\right]dv=0$$

BCs : $$\int_{\partial\Omega}\left[k\underline{\nabla}T.\underline{n}+hT+f\right]ds=0$$

Mesh : PDE : $$\sum_{i=1}^{Nm}\int_{\Omega_i}\left[\nabla k_i\underline{\nabla}T_i-\left(\rho c_p\right)_i\frac{\partial T_i}{\partial t}\right]dv=0$$

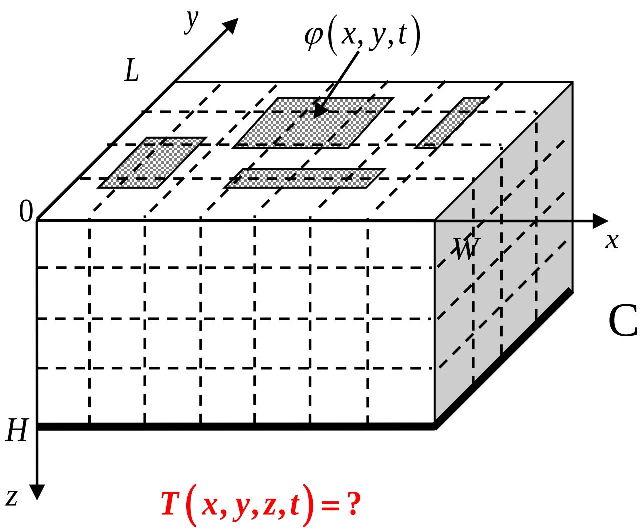
BCs : $$\sum_{i=1}^{Nm}\sum_{k}\int_{\partial\Omega_{ik}}\left[k_{ik}\underline{\nabla}T_i.\underline{n}_{ik}+h_{ik}T_i+f_{ik}\right]ds=0$$

Divergence th. PDE : $$\sum_{i=1}^{Nm}\int_{\partial\Omega_i}k_i\underline{\nabla}T_i\underline{n}_i ds-\int_{\Omega_i}\left(\rho c_p\right)_i\frac{\partial T_i}{\partial t}dv=0$$

$$\boxed{\sum_{j}k_{ij}\underline{\nabla}T_i\underline{n}_{ij}ds+\sum_{k}k_{ik}\underline{\nabla}T_i\underline{n}_{ik}ds} \Rightarrow \boxed{\sum_{j}k_i\underline{\nabla}T_i\underline{n}_{ij}ds-\sum_{k}\left[h_{ik}T_i+f_{ik}\right]ds}$$

Interior Exterior

I.2. Spatial Discretization, state representation



DHCP: we seek *T(x,y,z,t)*

$$\sum_{i=1}^{Nm}\int_{\partial\Omega_i}\left\{\sum_j k\,\underline{\nabla}T_i\,\underline{n}_{ij} - \sum_k\left[h_{ik}T_i + f_{ik}\right]\right\}ds - \int_{\Omega_i}\left(\rho c_p\right)_i\frac{\partial T_i}{\partial t}dv = 0$$

Cartesian coordinates (orth., direct) : $\underline{n}_{ij} = a_{ij}\underline{x} + b_{ij}\underline{y} + c_{ij}\underline{z}$

$$\underline{\nabla}T_i.\underline{n}_{ij} = \left(\frac{\partial T_i}{\partial x}a_{ij} + \frac{\partial T_i}{\partial y}b_{ij} + \frac{\partial T_i}{\partial z}c_{ij}\right)\Bigg|_{\partial\Omega_{ij}}$$

Finite Volumes: hyp: uniform values : -in cells Ω_i

-on boudaries $\partial\Omega_{ik}$

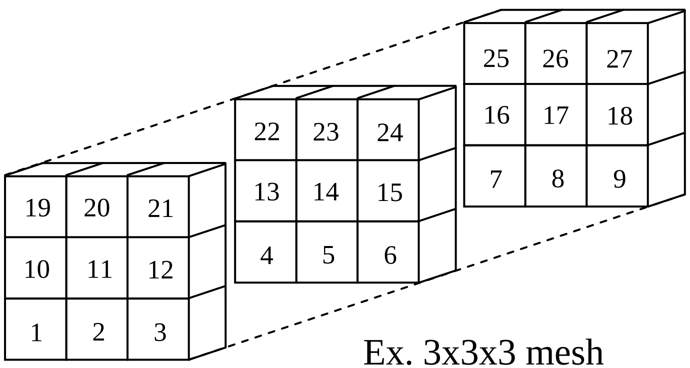
gradient approx.: virtual nodes : cell centers

$$\boxed{\sum_{i=1}^{Nm}\left[\sum_j S_{ij}k_{ij}\left\{\frac{a_{ij}\left[T_i - T_j\right]}{x_i - x_j} + \frac{b_{ij}\left[T_i - T_j\right]}{y_i - y_j} + \frac{c_{ij}\left[T_i - T_j\right]}{z_i - z_j}\right\} + \sum_k S_{ik}\left[h_{ik}T_i + f_{ik}\right] - \left(\rho c_p\right)_i V_i\frac{\partial T_i}{\partial t}\right] = 0}$$

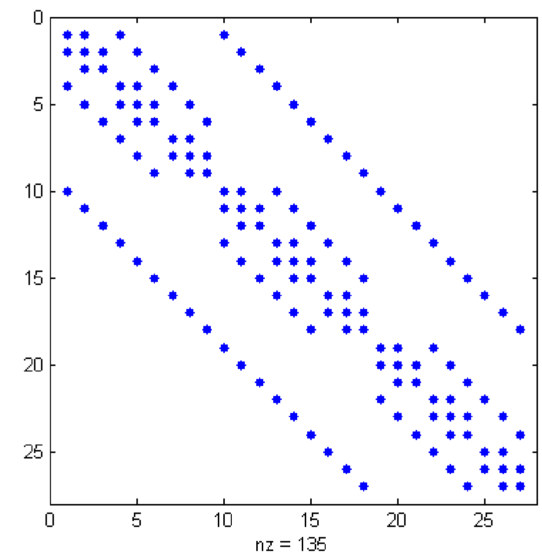
I.2. Spatial Discretization, state representation

$$\boxed{\sum_{i=!}^{Nm}\left[\sum_j S_{ij}k_{ij}\left\{a_{ij}\frac{T_i-T_j}{x_i-x_j}+b_{ij}\frac{T_i-T_j}{y_i-y_j}+c_{ij}\frac{T_i-T_j}{z_i-z_j}\right\}+\sum_k S_{ik}h_{ik}T_i+f_{ik}-(\rho c_p)_i V_i\frac{\partial T_i}{\partial t}\right]=0}$$

$$\boxed{\underline{\underline{A}}\,\underline{T}+\underline{\underline{B}}\,\underline{U}+\underline{\underline{K}}\,\underline{\dot{T}}=\underline{0}}$$



Ex. 3x3x3 mesh
Numbering cells with only one indice

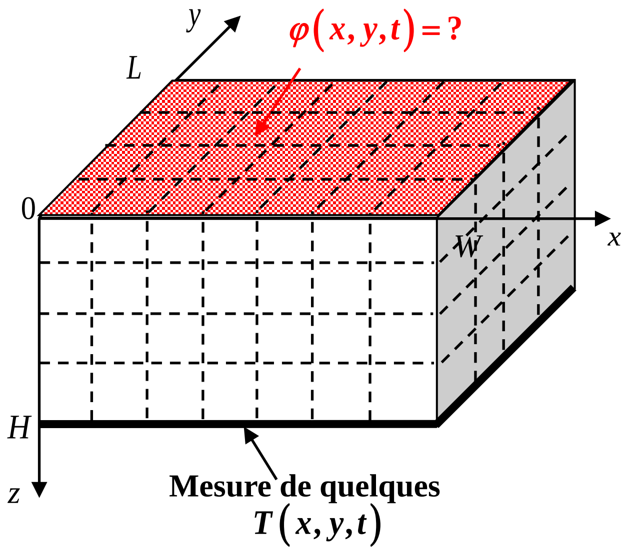


Elements position in matrix $\underline{\underline{A}}$

The key point to build these matrix is to have mesh tables (positions and connectivities)

DHCP: we seek $\underline{T}$

I.3. IHCP Description, a way to solve it



IHCP: we seek $\varphi(x,y,t)$

?

$$\underline{\underline{A}}\,\underline{T} + \underline{\underline{B}}_c\,\underline{U}_c + \underline{\underline{B}}_{nc}\,\underline{\varphi} + \underline{\underline{K}}\,\underline{\dot{T}} = \underline{0}$$

delay
+ deadening (amortissement)
+ parameters uncertaintie
+ noise

Ill-posed problem

regularization procedure : **Beck** + **Tikhonov**

I.3. IHCP Description, a way to solve it

Pseudo-inversion of state representation.

Calculated temperatures with DHCP

$$S = \sum_{n=1}^{Nn+1} \left\| \underline{Y}_n - \underline{Y}_n^* \right\|^2$$

Simulated or experimental measurements

$$\underline{T}_{n+1} = \underline{\underline{\mathbf{A}}}\,\underline{T}_n + \underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1} + \underline{\underline{\mathbf{B}}}_{\mathbf{nc}}\underline{\varphi}_{n+1}$$

$$\underline{Y}_{n+1} = \underline{\underline{C}}\,\underline{T}_{n+1}$$

Full implicite scheme needs matrix inversion

$\min(S)$

Pseudo inverse

$$\underline{\hat{\varphi}}_{n+1} = \left[\underline{\underline{D}}^T \underline{\underline{D}}\right]^{-1} \underline{\underline{D}}^T \left[\underline{Y}_{n+1}^* - \underline{b}_{n+1}\right]$$

Solution in least mean square sens = quasi-solution

$\underline{\underline{D}} = \underline{\underline{C}}\,\underline{\underline{\mathbf{B}}}_{\mathbf{nc}}$ Dynamical sensibilitiy

$$\underline{b}_{n+1} = \underline{\underline{C}}\,\underline{\underline{\mathbf{A}}}\,\underline{T}_n + \underline{\underline{C}}\,\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1}$$

I.4. Stabilization / regularization techniques

Two combined techniques: 1. Function specification: futur times technique

Temporary assumption on the sought values evolution

$$S=\sum_{n=1}^{Nn+1}\sum_{f=0}^{Nf}\left\|\underline{Y}_{n+f}-\underline{Y}^{*}_{n+f}\right\|^{2}$$

$\forall f=1,2..Nf$ we assume $\underline{\varphi}_{n+f}=\underline{\varphi}_{n}$

New parameter

$$f=1:\quad \underline{Y}_{n+1+1}=\underline{\underline{C}}\left[\underline{\underline{\mathbf{A}}}\,\underline{T}_{n+1}+\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1+1}+\underline{\underline{\mathbf{B}}}_{\mathbf{nc}}\underline{\varphi}_{n+1+1}\right]$$

$$\underline{Y}_{n+1+1}=\underline{\underline{C}}\left[\underline{\underline{\mathbf{A}}}\left[\underline{\underline{\mathbf{A}}}\,\underline{T}_{n}+\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1}+\underline{\underline{\mathbf{B}}}_{\mathbf{nc}}\underline{\varphi}_{n+1}\right]+\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1+1}+\underline{\underline{\mathbf{B}}}_{\mathbf{nc}}\underline{\varphi}_{n+1+1}\right]$$

Repeat for $f=2\ldots Nf$: $\Rightarrow$

$$\underline{\hat{\varphi}}_{n+1}=\left[\underline{\underline{\mathbf{D}}}^{T}\underline{\underline{\mathbf{D}}}\right]^{-1}\underline{\underline{\mathbf{D}}}^{T}\left[\underline{\mathbf{Y}}^{*}_{n+1}-\underline{\mathbf{b}}_{n+1}\right]$$

$$\underline{\mathbf{Y}}^{*}_{n+1}=\begin{bmatrix}\underline{Y}^{*}_{n+1}\\ \underline{Y}^{*}_{n+1+1}\\ \vdots\\ \underline{Y}^{*}_{n+1+f}\\ \vdots\\ \underline{Y}^{*}_{n+1+Nf}\end{bmatrix}\qquad \underline{\underline{\mathbf{D}}}=\begin{bmatrix}\underline{\underline{\mathbf{D}}}_{n+1}\\ \underline{\underline{\mathbf{D}}}_{n+1+1}\\ \vdots\\ \underline{\underline{\mathbf{D}}}_{n+1+f}\\ \vdots\\ \underline{\underline{\mathbf{D}}}_{n+1+Nf}\end{bmatrix}\qquad \underline{\mathbf{b}}_{n+1}=\begin{bmatrix}\underline{\mathbf{b}}_{n+1}\\ \underline{\mathbf{b}}_{n+1+1}\\ \vdots\\ \underline{\mathbf{b}}_{n+1+f}\\ \vdots\\ \underline{\mathbf{b}}_{n+1+Nf}\end{bmatrix}$$

$$\underline{\underline{\mathbf{D}}}_{n+1+f}=\underline{\underline{C}}\sum_{j=0}^{j=f}\underline{\underline{\mathbf{A}}}^{j}\underline{\underline{\mathbf{B}}}_{\mathbf{nc}}$$

$$\underline{\mathbf{b}}_{n+1}=\underline{\underline{C}}\left[\underline{\underline{\mathbf{A}}}^{f+1}\underline{T}_{n}+\sum_{j=0}^{f}\underline{\underline{\mathbf{A}}}^{j}\underline{\underline{\mathbf{B}}}_{\mathbf{c}}\underline{U}_{c,n+1+f-j}\right]$$

I.4. Stabilization / regularization techniques

2. Tikhonov penalisation: adding informations on the solution

$$S=\sum_{n=1}^{Nn+1}\left[\sum_{f=0}^{Nf}\left\|Y_{n+f}-Y^{*}_{n+f}\right\|^{2}+\mu_{T}\left\|\underline{\underline{R}}\,\underline{\varphi}_{n}\right\|^{2}\right]$$

New parameter

$\min(S)$

$$\underline{\hat{\varphi}}_{n+1}=\left[\underline{\underline{\mathbf{D}}}^{T}\underline{\underline{\mathbf{D}}}+\mu_{T}\underline{\underline{R}}^{T}\underline{\underline{R}}\right]^{-1}\underline{\underline{\mathbf{D}}}^{T}\left[\underline{\mathbf{Y}}^{*}_{n+1}-\underline{\mathbf{b}}_{n+1}\right]$$

0 order : $\underline{\underline{R}}=\underline{\underline{I}}$

1 order : $\underline{\underline{R}}$ derivates $\underline{\varphi}$ one time in space directions

2 order : $\underline{\underline{R}}$ derivates $\underline{\varphi}$ twice in space directions

I.4. Stabilization / regularization techniques

Indicators

Standard deviations:

Heat flux criterion (available in testing stage)

$$\sigma_{\varphi} = \sqrt{\left[\frac{1}{(Nn - Nf)\ Npnc} \sum_{n=1}^{(Nn-Nf)} \sum_{pnc=1}^{Npnc} \left[\hat{\varphi}_{pnc,n} - \varphi_{pnc,n} \right]^2 \right]}$$

Temperature criterion (useful for the stabilisation procedure)

$$\sigma_{Y} = \sqrt{\left[\frac{1}{(Nn - Nf)\ Nq} \sum_{n=1}^{(Nn-Nf)} \sum_{q=1}^{Nq} \left(\hat{Y}_{q,n} - Y^{*}_{q,n} \right)^2 \right]}$$

Criterion to choose the time step calculation:

$$Fo_{\Delta t} = \frac{a\Delta t}{d^2} > 0.1$$

Allow to have good sensibility

I.4. Stabilization / regularization techniques

Regularization procedure: discrepancy principle

Choice of *Nf* (time direction)

Increasing *Nf* till :

$$\boxed{\sigma_Y \approx \sigma_{Y^*}/2}$$

Choice of μ_T (spaces directions)

Iterative calculation till respecting
the discrepancy principle:

$$\boxed{\sigma_Y \approx \sigma_{Y^*}}$$

Searching method: dichotomie
Approximately 10 iterations

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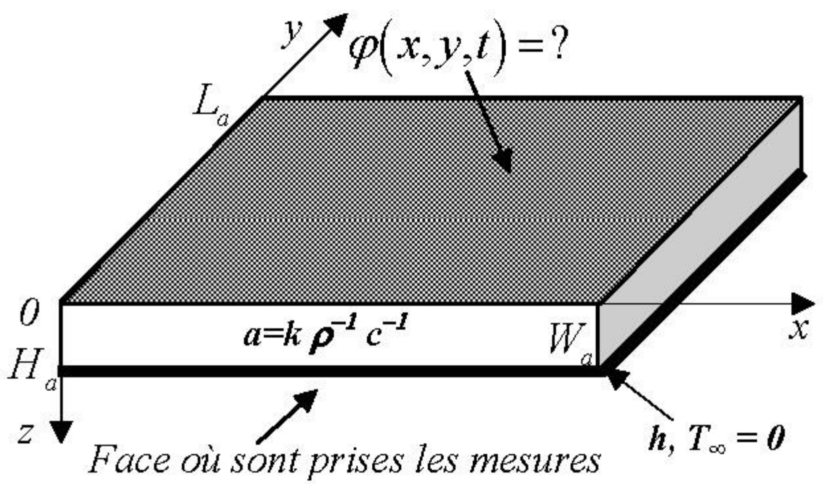
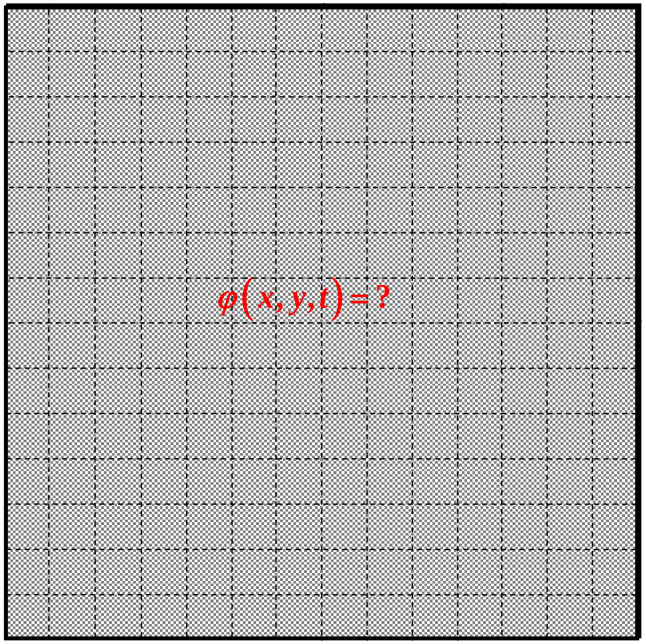
II. Numerical validation with simulated data.

Description of the numerical test



$$H_a = 4[mm]$$

Test case description



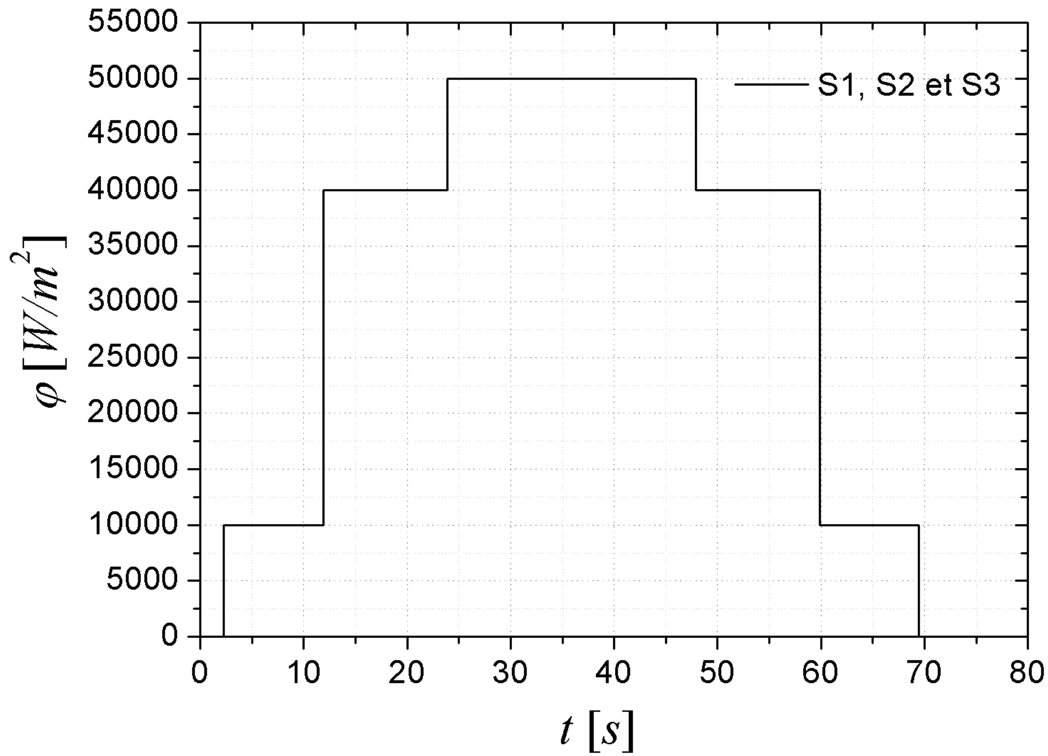
Heaters location at z = 0 ([*cm*])

Convection coefficient are estimated using correlation

Steel plate $k \approx 70\left[W.m^{-1}.K^{-1}\right]$ $\rho \approx 7850\left[kg.m^{-3}\right]$ $c_p \approx 480\left[J.kg^{-1}.K^{-1}\right]$

II. Numerical validation with simulated data.

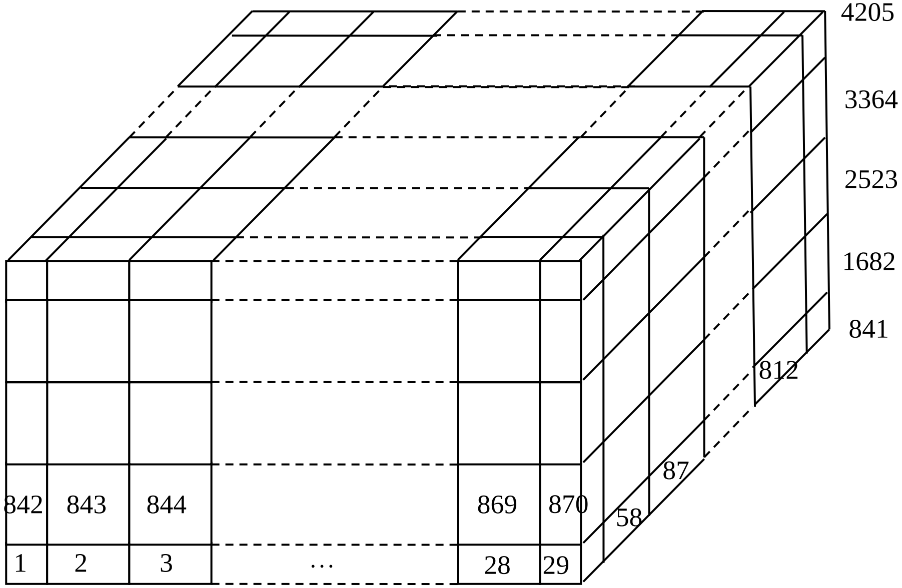
Description of the numerical test: Heating Scenario



teme evolution of heat flux.

II.Numerical validation with simulated data.

Choosing mesh



Sketch of the mesh with node numbering : 841 x *Nn* inconnues

Δx [mm]	Δy [mm]	Δz [mm]	nombre de volumes	Nombre d'inconnues à l'instant n
5	5	1	4205	841

Mesh (plate thickness $Ha = 4[mm]$).

Choosing time step: $\Delta t = 0.24[s]$
(*Fo* criterion 0.3)

II. Numerical validation with simulated data.

Matlab is used to implemente all the equations presented in previous slides

Results :

Simulated noise

$\sigma_{Y^*} = 0.0186$ [C].

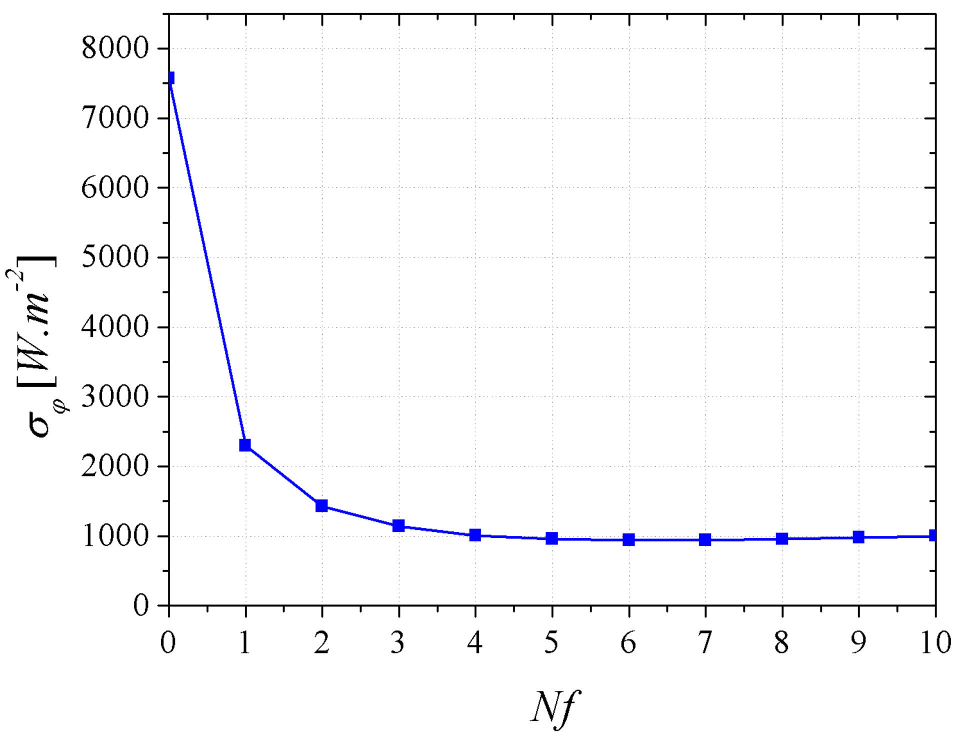
Nf	μ_{Ti}	Cond $\left[\underline{\underline{\mathbf{D}}}^T\underline{\underline{\mathbf{D}}} + \mu_{Ti}\underline{\underline{R}}^T\underline{\underline{R}}\right]$	Max $\underline{X}_{d,pnc,q}$	Max cor_d	σ_{Y^*}	σ_Y	σ_φ	Temps de calculs du PICC
-	-	-	[°C.m²/W]	-	[°C]		[W/m²]	[s]
0	0	20.9	4.48E-6	0.538	0	1.80E-15	1.53E-9	225.3
					0.0186	1.65E-12	7.57E3	224.05
1	0	35.6	9.71E-6	0.615	0.0186	0.0138	2.30E3	846.5
	$\mu_{T0} = 3.13\text{E-}11$	13.34				0.0186	1.49E3	6903
	$\mu_{T1} = 1.01\text{E-}10$	33.12					1.42E3	9192
	$\mu_{T2} = 5.29\text{E-}11$	10.45					1.44E3	7669

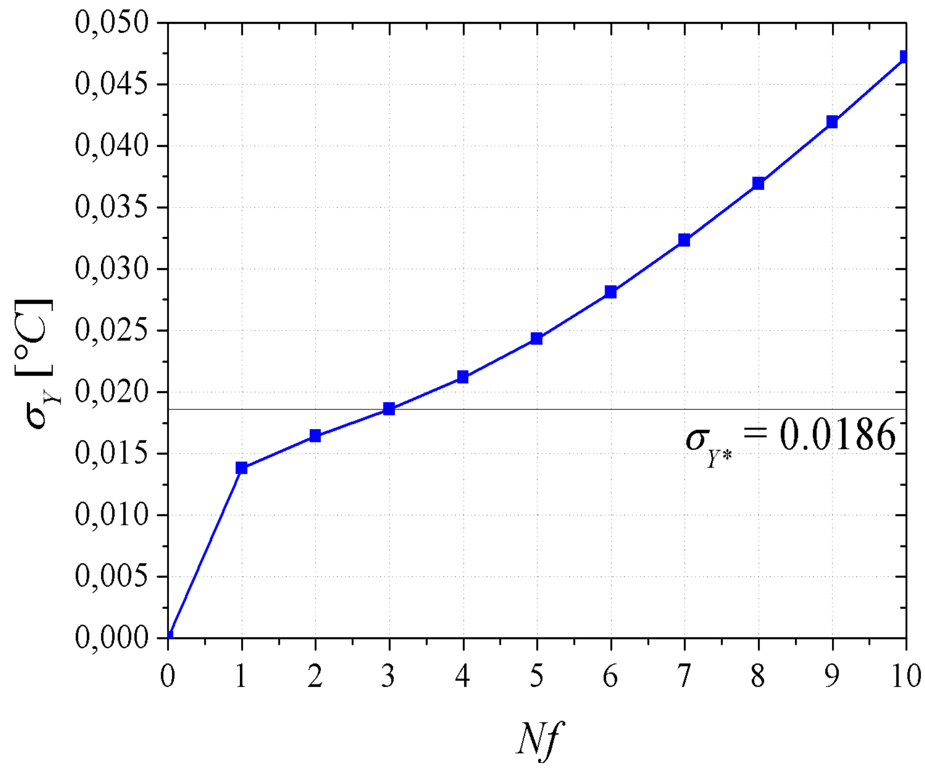
Numerical case results : Ha = 4[mm], Δt= 0.24[s].

II. Numerical validation with simulated data.

Results : influence of *Nf*

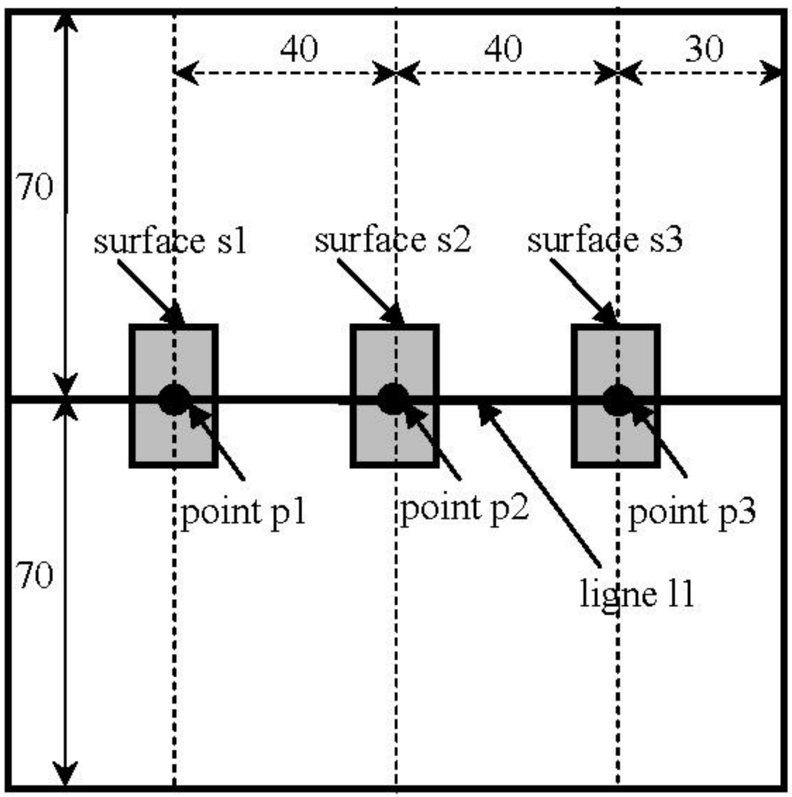
$$\boxed{\sigma_{Y^*} = 0.0186 \quad [\ \mathrm{C}].}$$



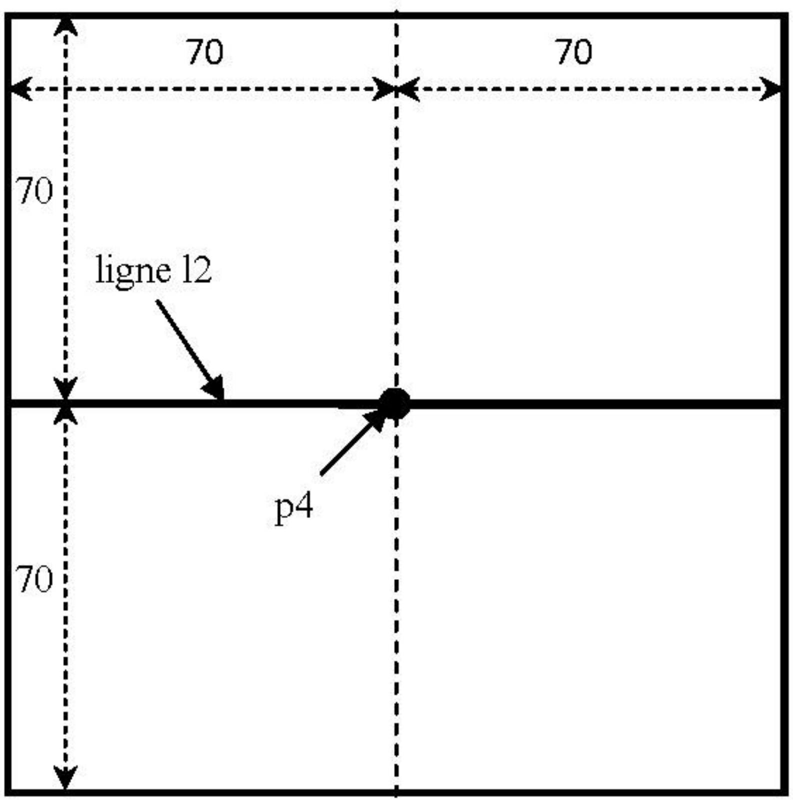


II. Numerical validation with simulated data.

Geometrical objects for graphical presentation of results



For heat flux at $z = 0$.

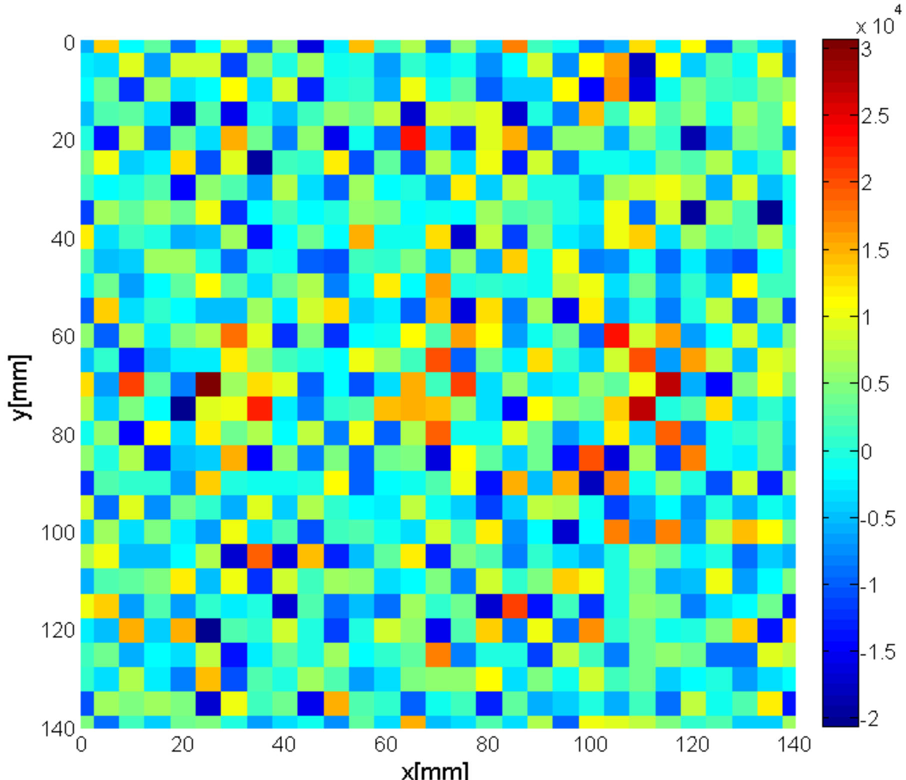


For temperatures at $z = Ha$.

II. Numerical validation with simulated data.

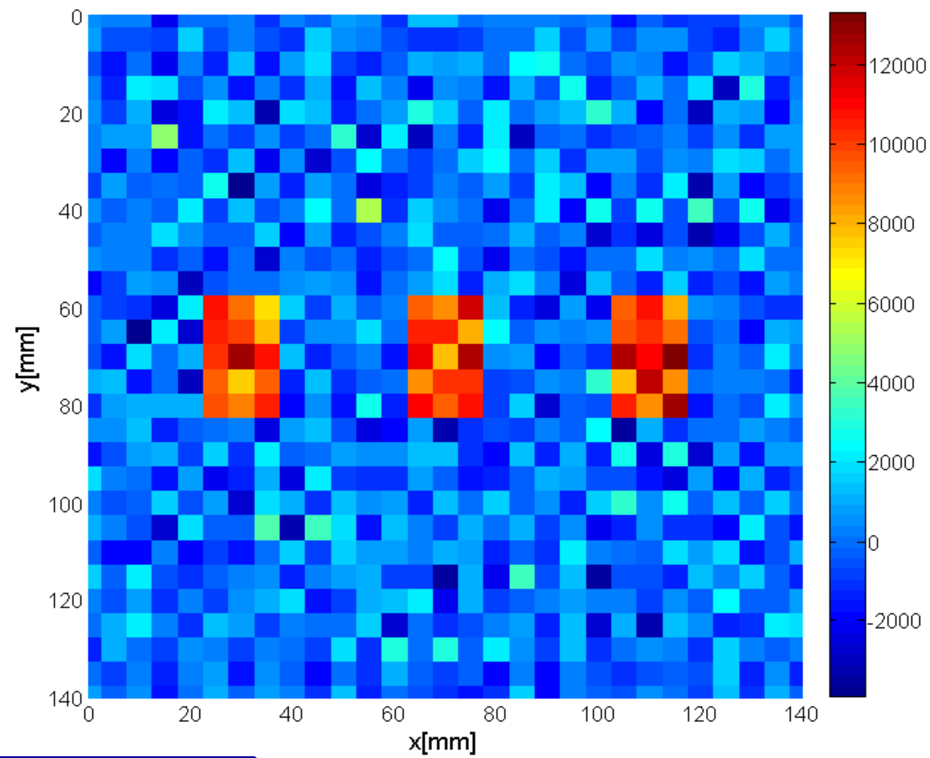
Results:

Estimated heat flux at $t = 4.8[s]$



without régularisation.

$$\sigma_{Y^*} = 0.0186 \quad [\ C].$$

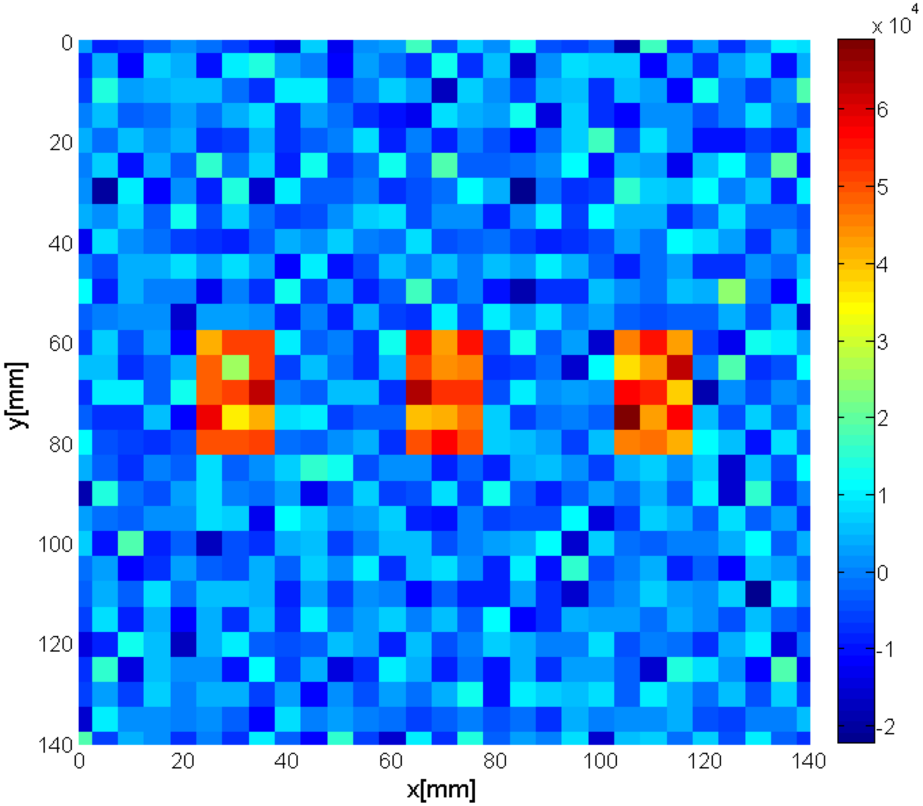


with régularisation.

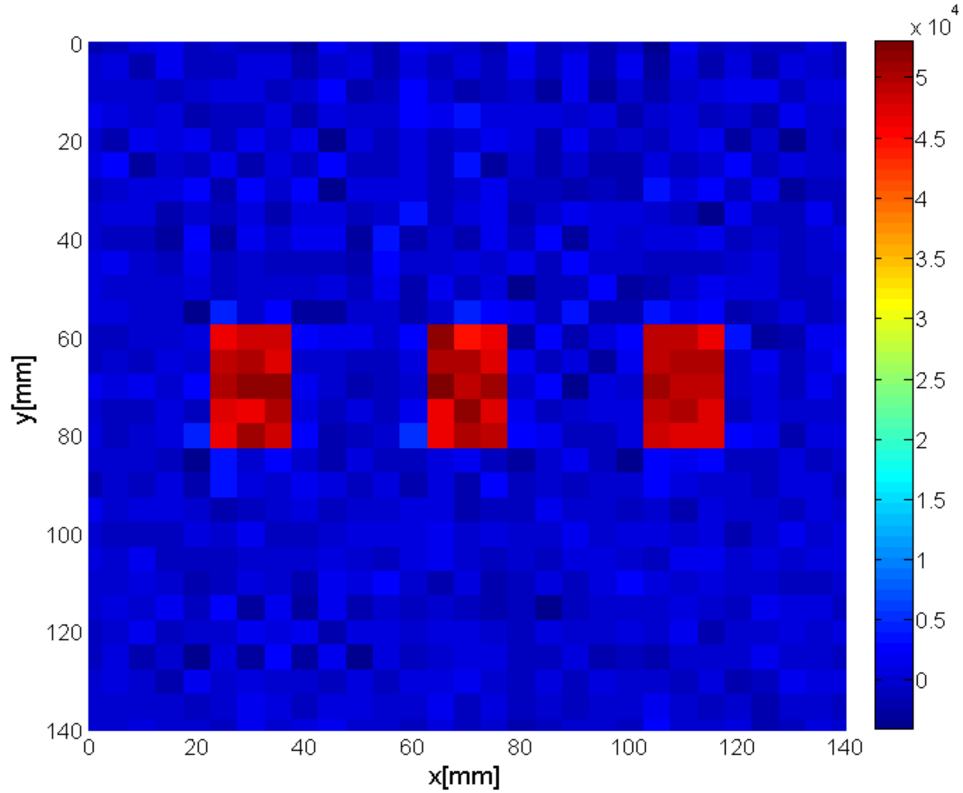
II. Numerical validation with simulated data.

Results :

Estimated heat flux at $t = 36[s]$



Without regularisation.

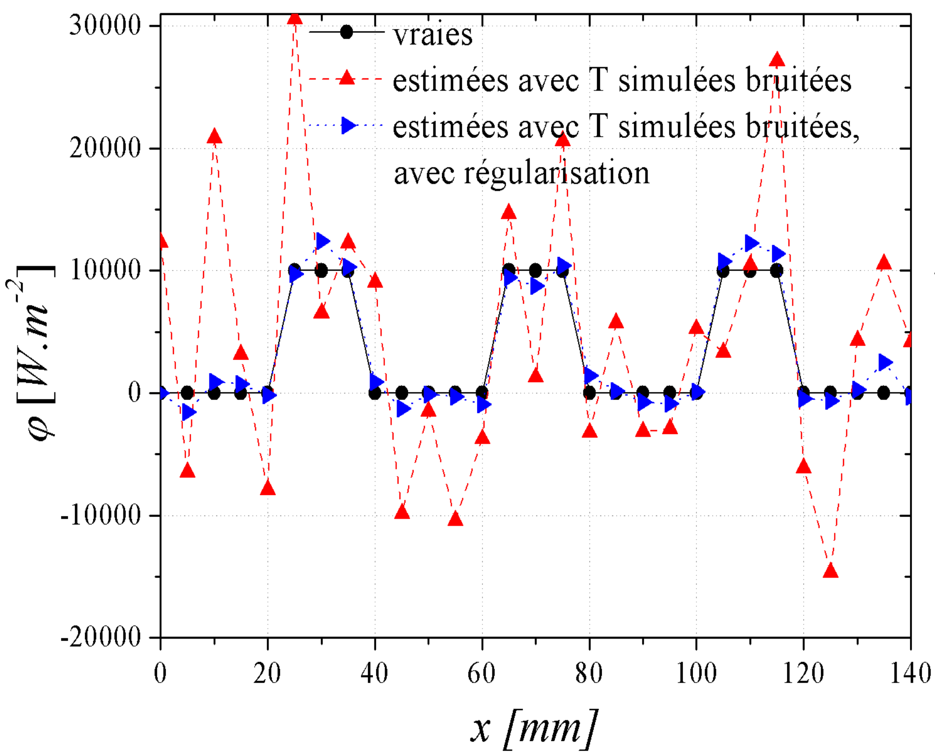


With regularisation.

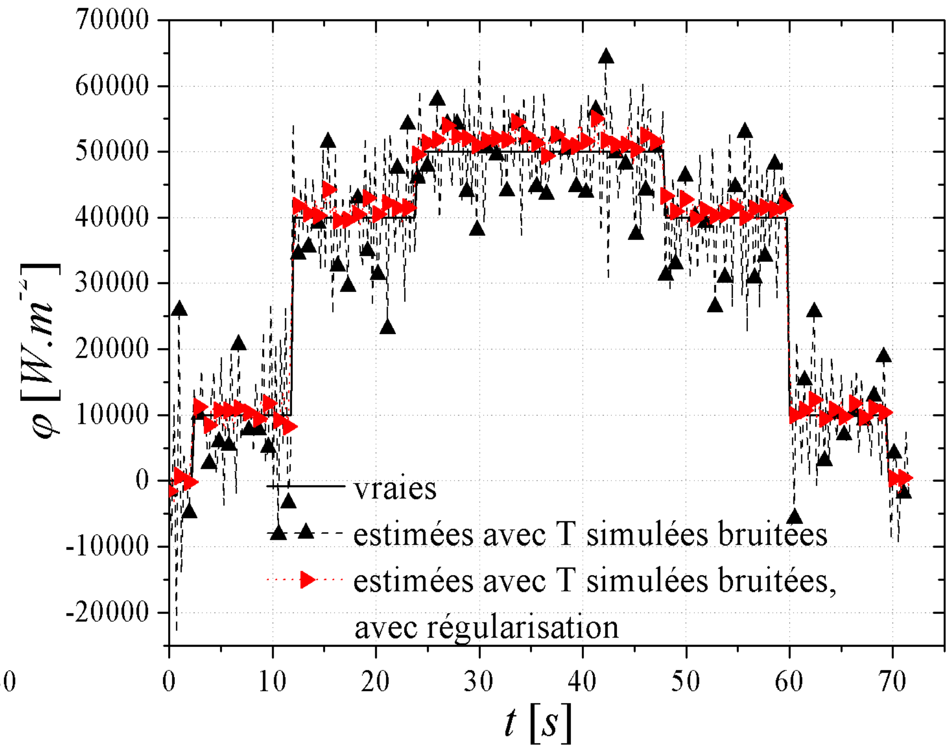
$\sigma_{Y^*} = 0.0186$ [C].

II. Numerical validation with simulated data.

Results :



Estimated heat flux on line l1 at t = 4.8[s].



Heat flux evolution at point p2.

II. Numerical validation with simulated data.

Numerical tests with simulated data

Calculation of criterion σ_{φ} et σ_{T}
+ time step choice (Fourier)

good results

Experimental test

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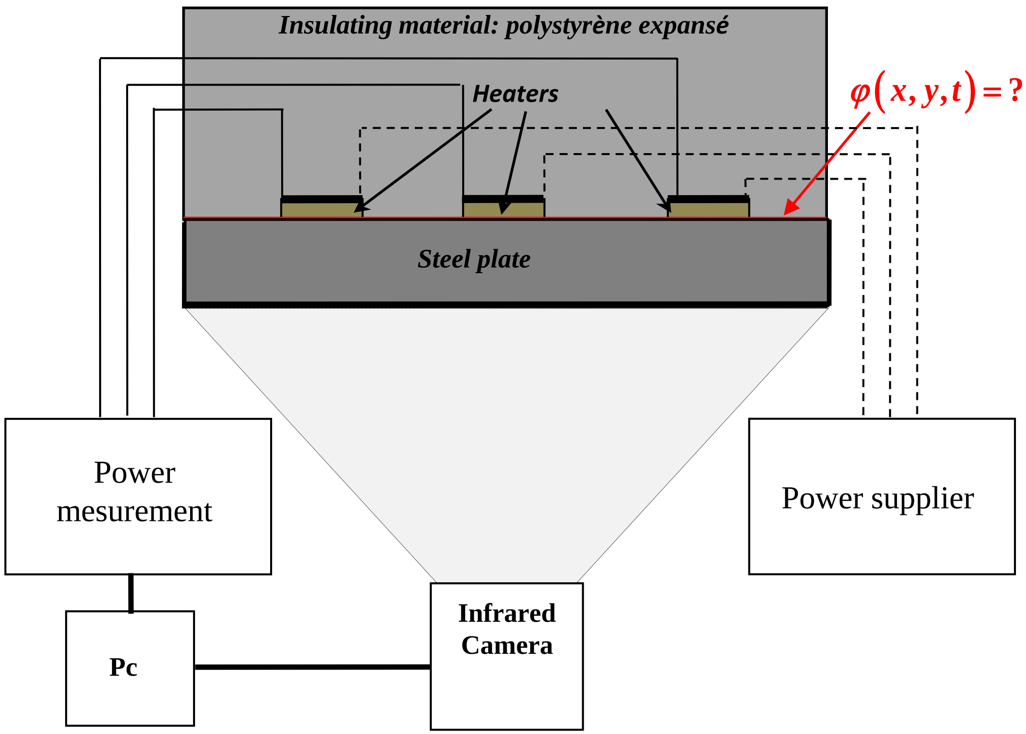
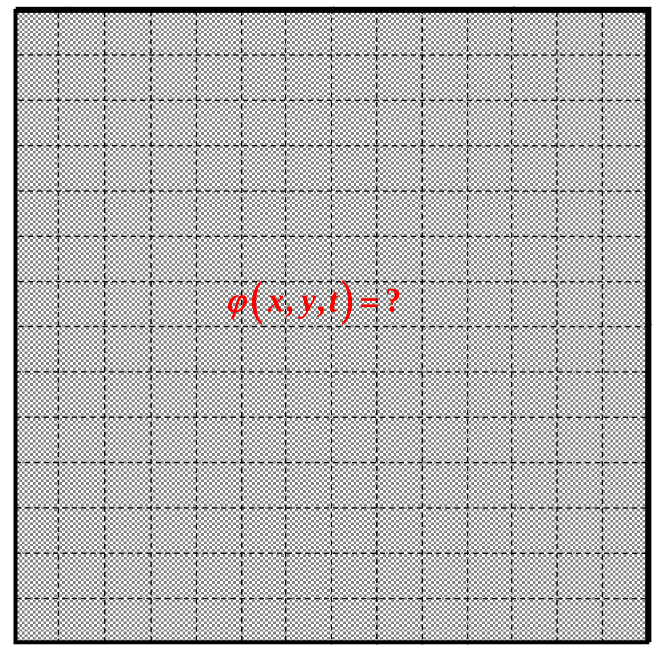


Sketch of the experimental bench



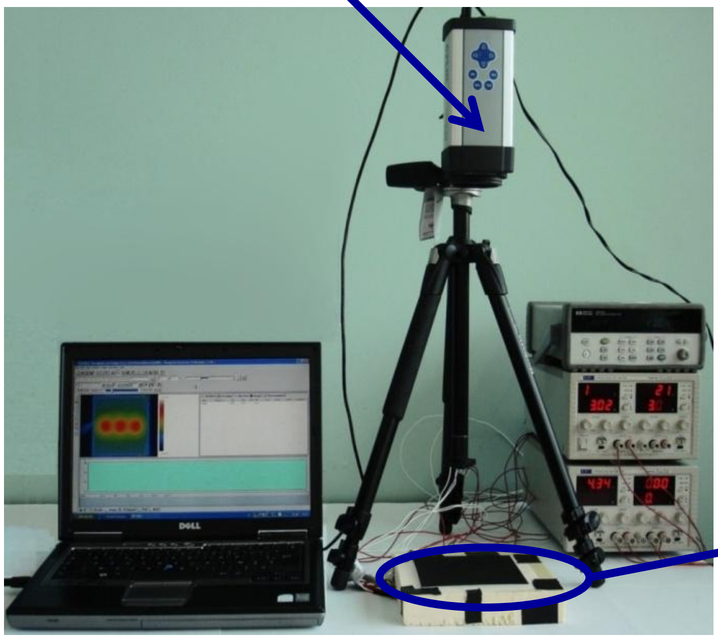
Heaters location (cm)

We suppose that the intensity location is unkown

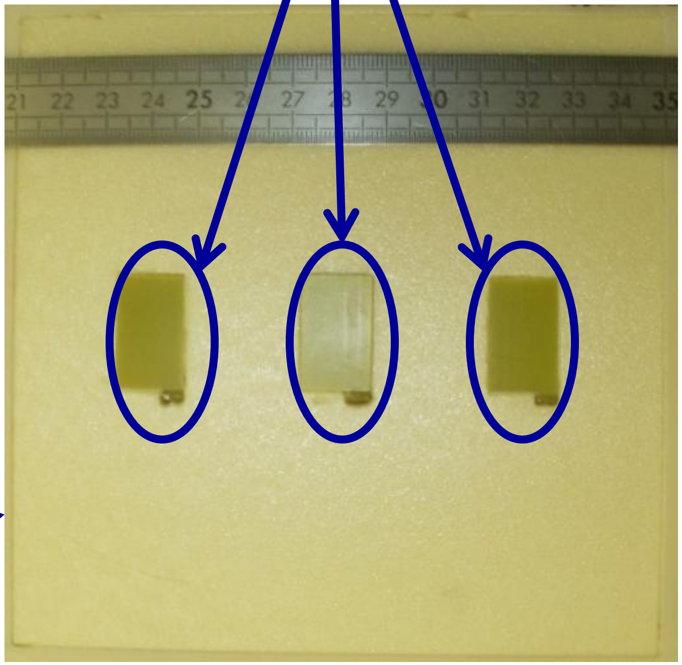
III. Experimental bench

Infrared camera

Heaters



Picture of devices of the experimental bench



Picture of heaters in the insulating material

Δx [mm]	Δy [mm]	Δz [mm]	nombre de volumes	Nombre d'inconnues à l'instant n
5	5	1	4205	841

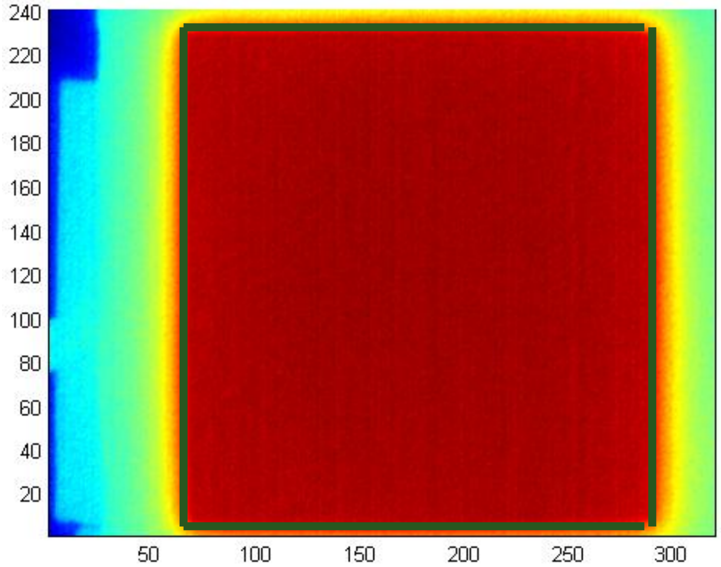
Steel plate

$$\rho \approx 7850\left[kg.m^{-3}\right]$$
$$c_p \approx 480\left[J.kg^{-1}.K^{-1}\right]$$
$$k \approx 70\left[W.m^{-1}.K^{-1}\right]$$

III. Experimental bench

Measures treatment :

An OS field

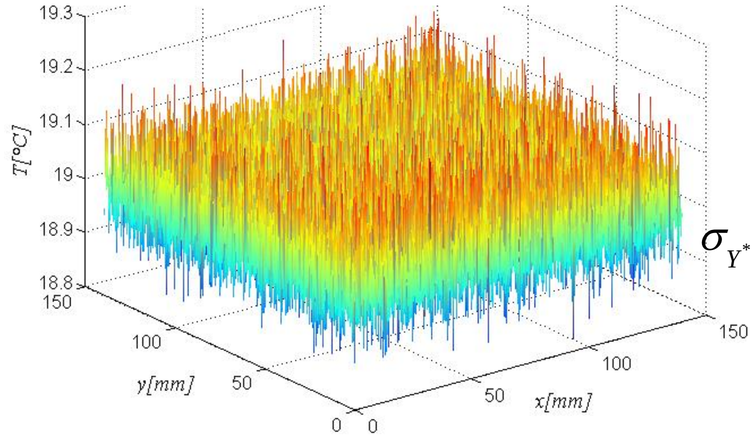


1) Plate cutting out all pictures

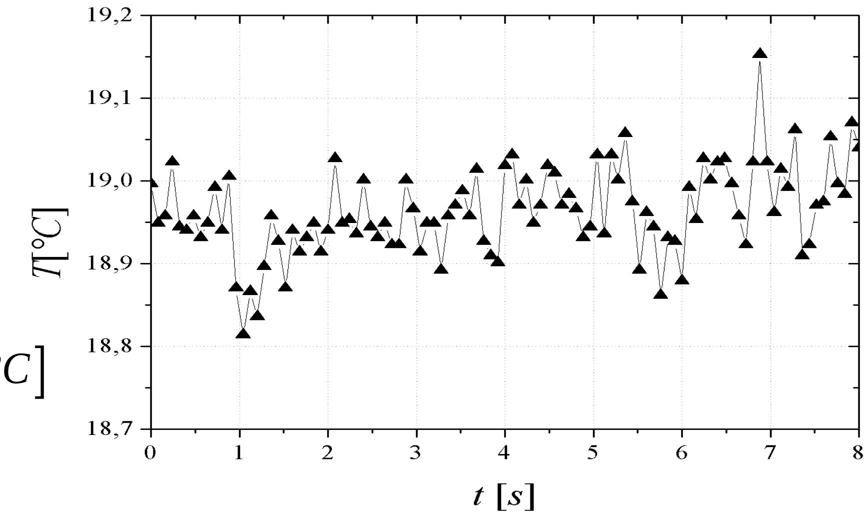
2) Subtracting the first picture to all others and adding the average of the first picture

III. Experimental bench

Measures treatment : 3) Sliding average

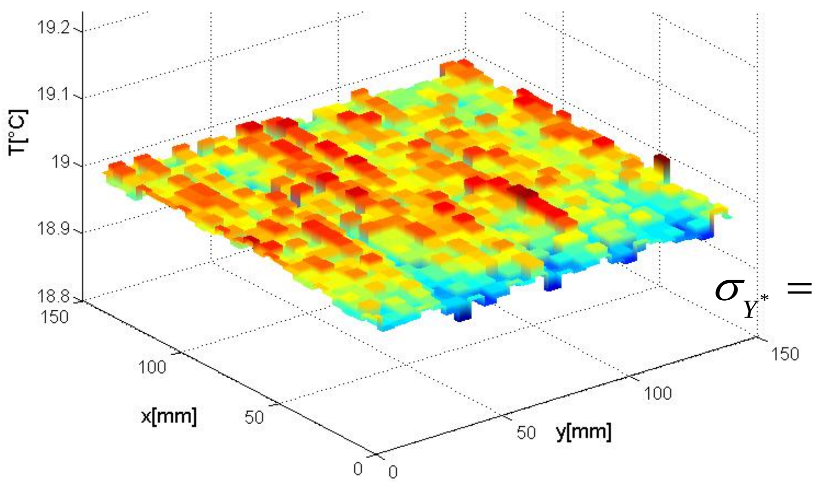


$$\sigma_{Y^*} = 0.083 \left[{}^\circ C\right]$$

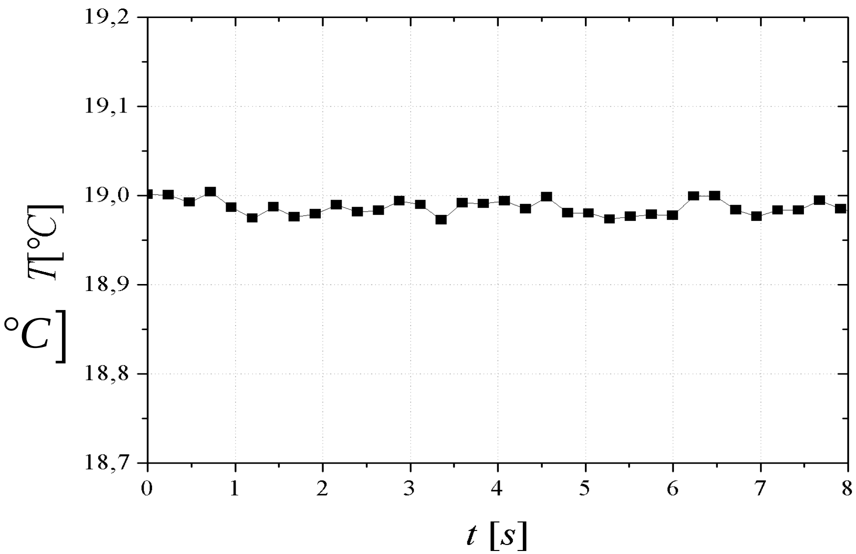


Measured temperatures at the center f = 12.5[*Hz*] Δt = 0.08[*s*].

210x210pixels (1 pixel : 0.44*mm*²)



$$\sigma_{Y^*} = 0.0186\left[{}^\circ C\right]$$



Treated temperatures at the center point, Δt = 0.24[*s*].

29x29 areas (1 area : 25*mm*² : 50 pixels)

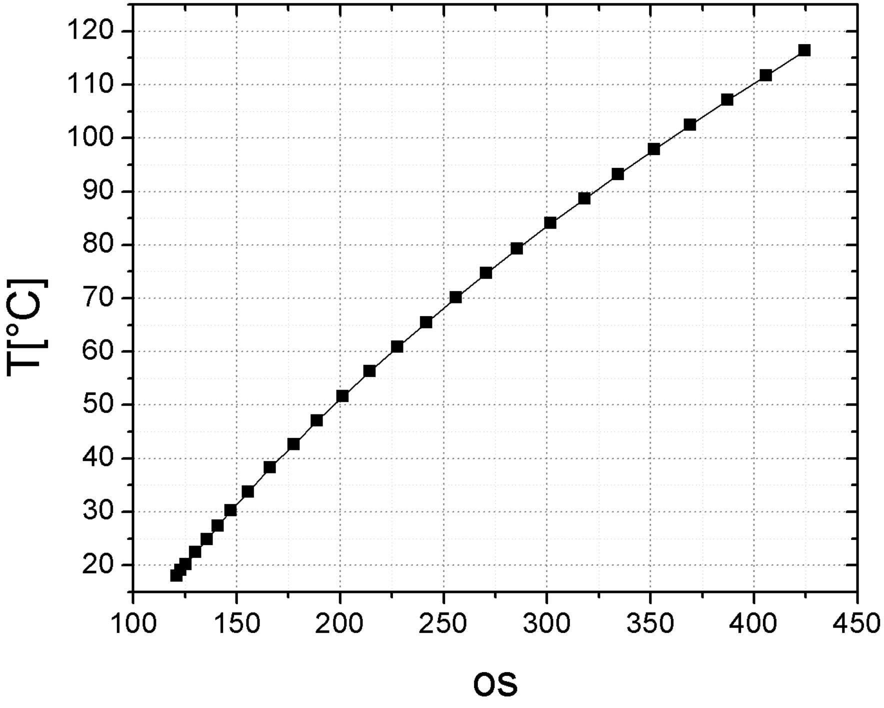
III. Experimental bench

Measures treatment : 4)use the calibration function to get temperatures

In situ calibration procedure

soldered thermocouples

$$T\left(os\right)=8,529881E^{-7}os^{3}-1,050853E^{-3}os^{2}+6,872698E^{-1}os-51,11124$$



same material, same black paint
across a regulated furnace

III. Experimental bench

Inversion results

Nf	μ_{Ti}	Cond $\left[\underline{\underline{D}}^T\underline{\underline{D}}+\mu_{Ti}\underline{\underline{R}}^T\underline{\underline{R}}\right]$	Max $\underline{X}_{d,pnc,q}$	Max cor_d	σ_Y	Temps de calculs du PICC
-	-	-	[°C m2/W]	-	[°C]	[s]
0	0	20.9	4.48E-6	0.538	1.88e-15	230.35
1		35.6	9.71E-6	0.615	0.0083	871.44
2	0	55.6	1.43E-5	0.674	0.0099	1382
	μ_{T0} =1.45E-10	12.3			0.0186	11579
	μ_{T1} =5.12E-10	48.2				15406
	μ_{T2} =1.06E-9	8.21				17975

Experimental resuls

increase Nf till :

$$\boxed{\sigma_Y \approx \sigma_{Y^*}/2}$$

choosing μ_T :

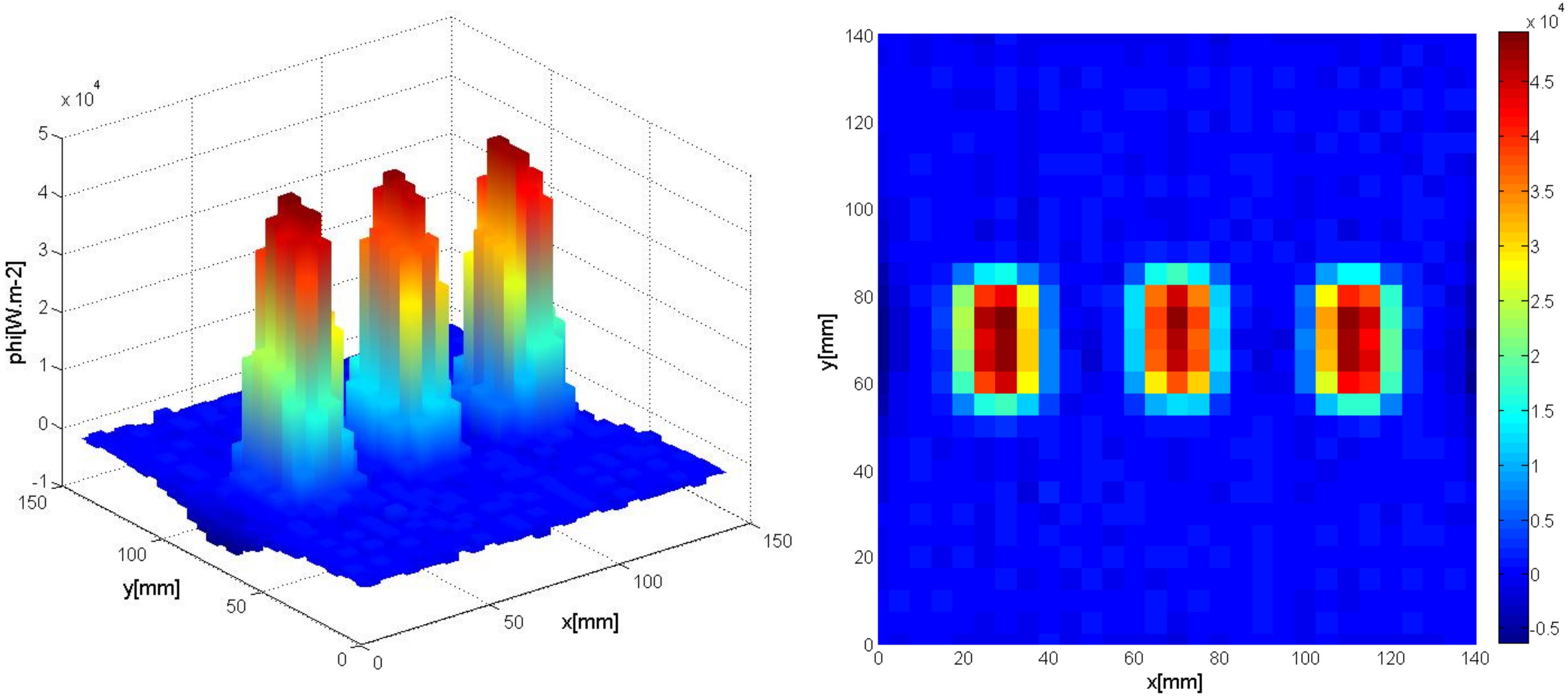
$$\boxed{\sigma_Y \approx \sigma_{Y^*}}$$

dichotomie (≈10 iterations)

III. Experimental bench

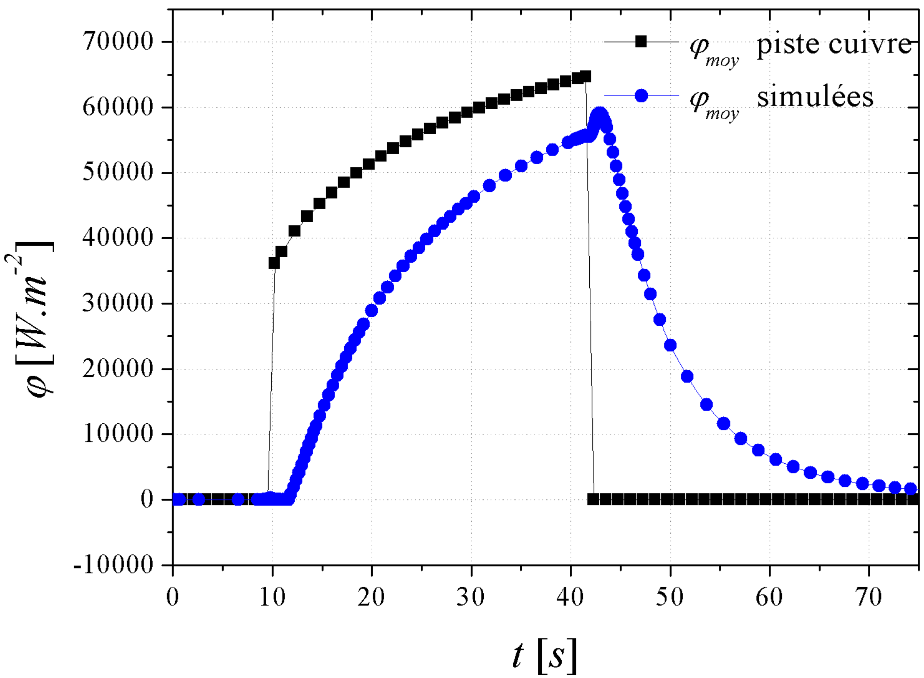
Inversion results

Heat flux estimated with regularization, at t=36[s].

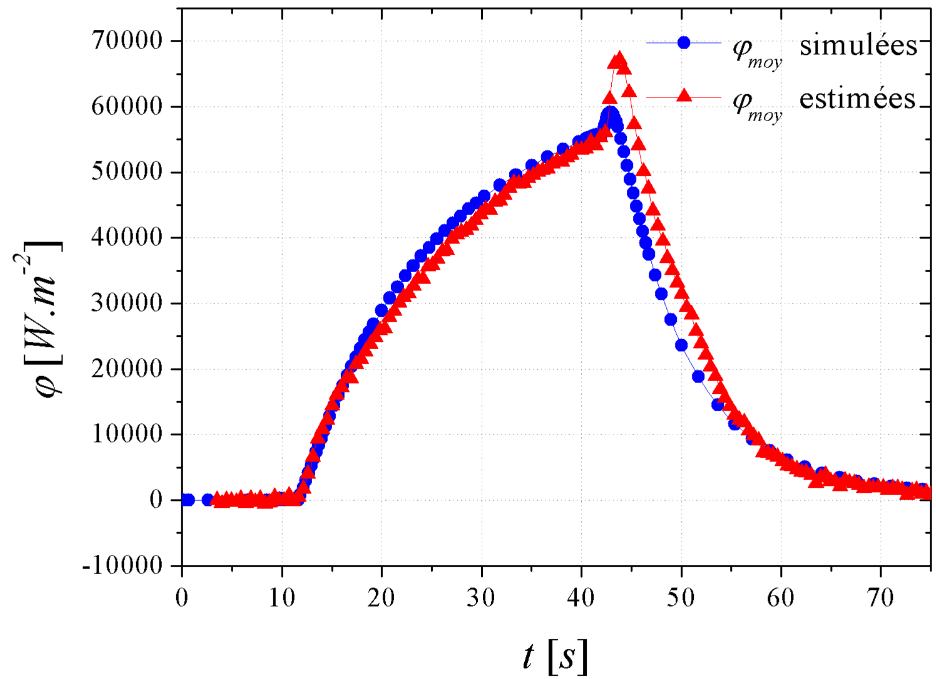


III. Experimental bench

Inversion results



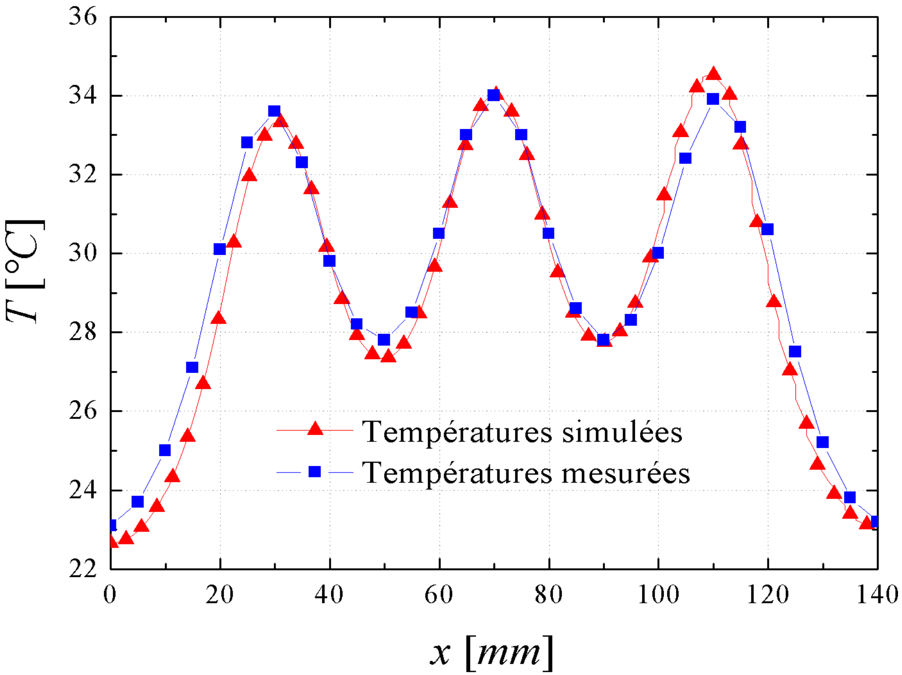
Evolution of simulated average heat flux on S2 and average heat flux measured for S2.



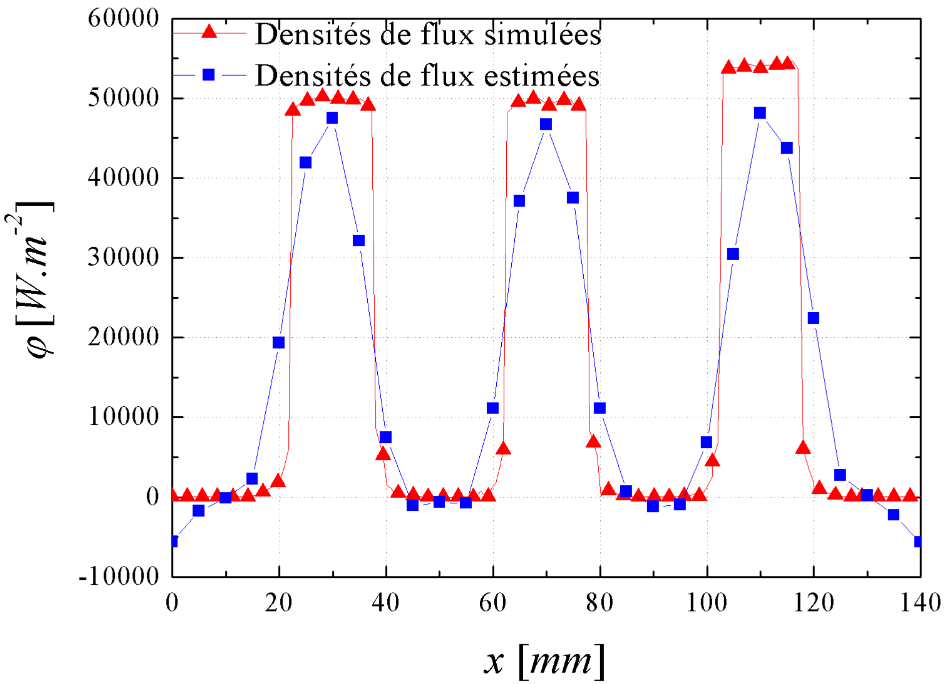
Evolution of simulated and estimated average heat flux on S2 (with regularisation)

III. Experimental bench

Inversion results



Simulated and measures temperatures on line l2 at t = 36[s].

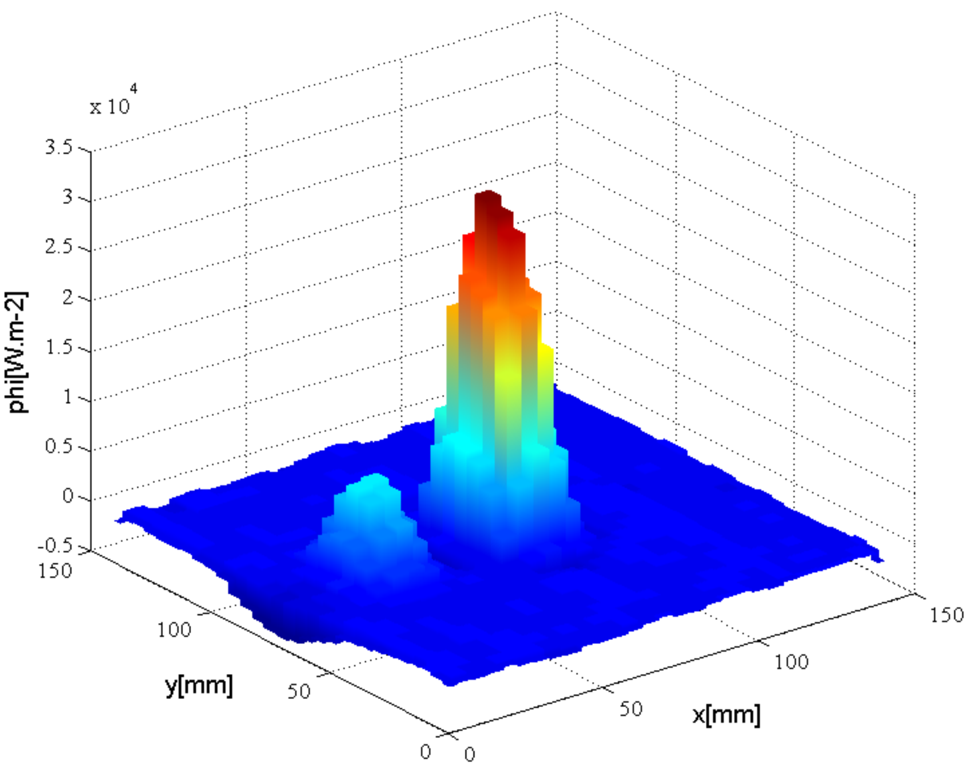


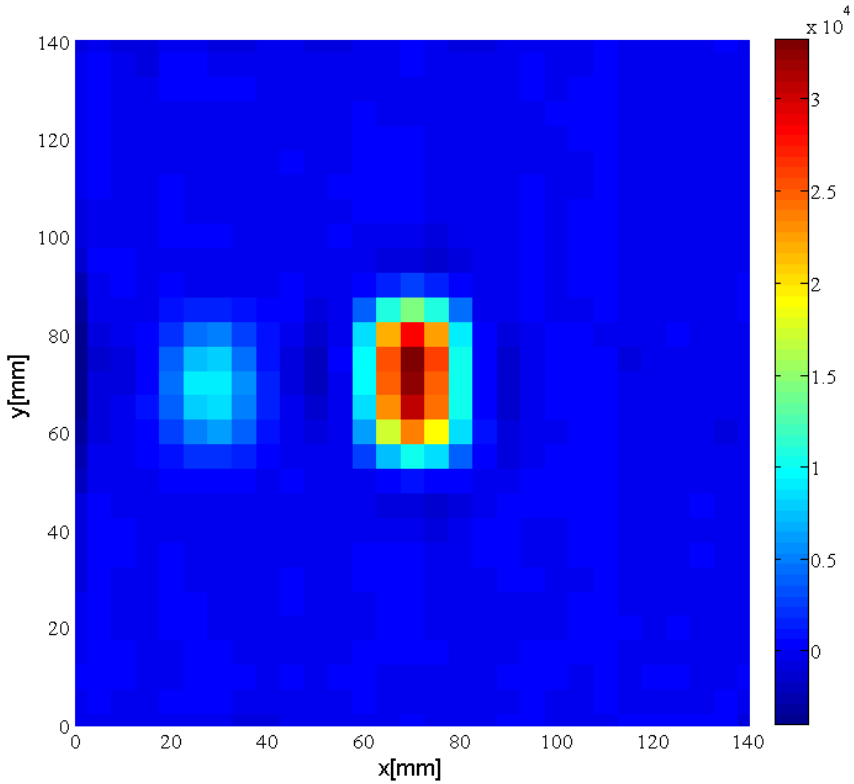
Simulated and estimated heat flux on line l1 à t = 36[s].

III. Experimental bench

Other inversion results

Heat flux estimated with regularization, another heating scenario.





Conclusion

Non iterative method:
good results

But:

Inversion of matrix of size N*m : long calculation time*
Storage of full matrix of size *Nm* : need a lot of storage space

Limited spatiale resolution

Limitations overrunning : -iterative methods with gradient
-zero order methods…

End

Thank you for your attention