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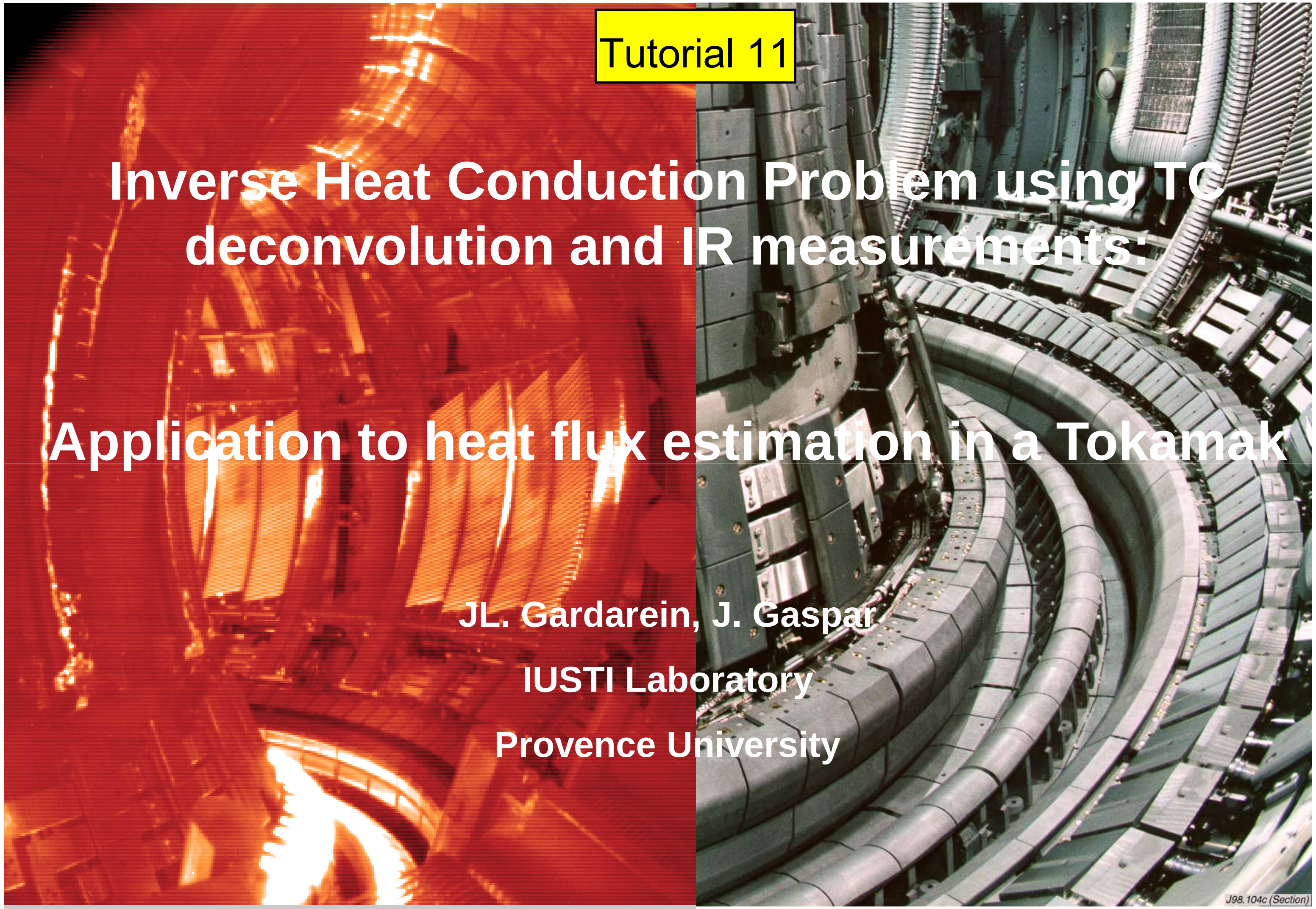
Tutorial 11



Inverse Heat Conduction Problem using TC deconvolution and IR measurements:

Application to heat flux estimation in a Tokamak

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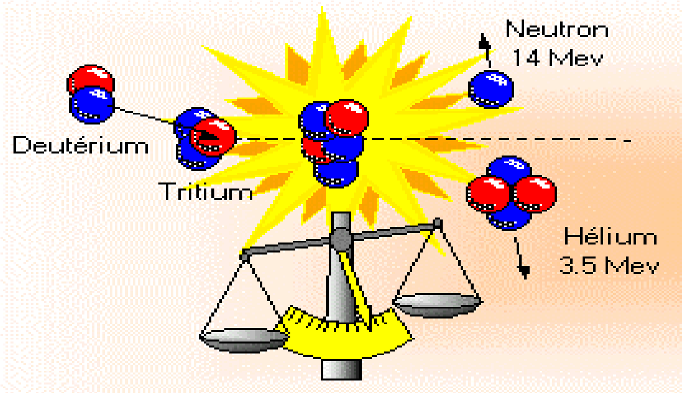
Summarize

1. Experimental Set-Up Problematic

2. Description of the method
   Application to a 1D Inverse Heat Conduction Problem

3. Application to a 2D experimental case
   (If we have time)

Plasma at T ~ 100 millions of degrés
Lawson Critera:
(Density) x (T°) x (Confinement Time) > 10^{21} KeV.s.m^{-3}



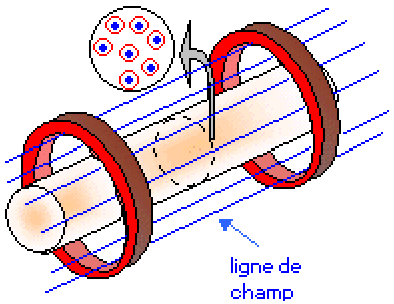
The tokamaks…

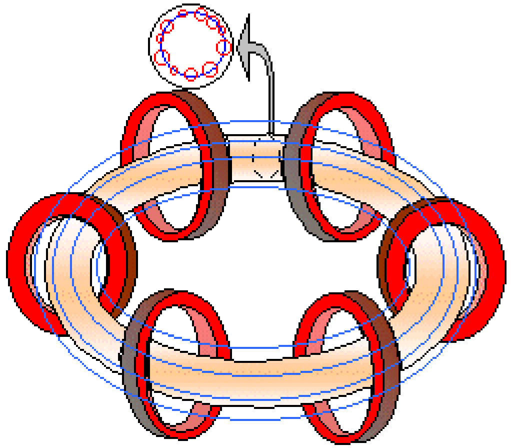
The confinement is insuring with 2 magnetic fields :

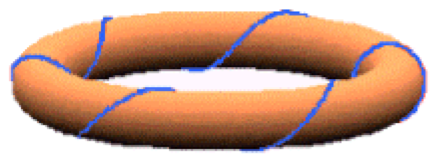
– an axial field produced with the toroidal coils

– a poloidal field created with the plasma current

The resulting magnetic fields are helicoïdal







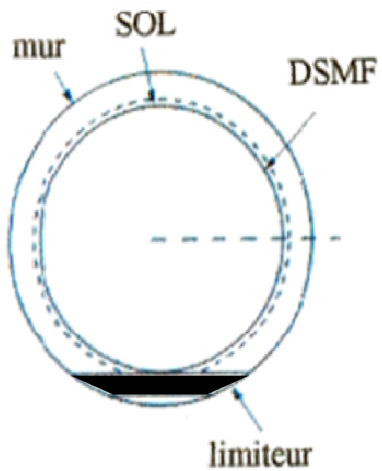
Plasma Wall Interaction: Heat flux of about 10MW/m2

DSMF : Dernière surface magnétique fermée

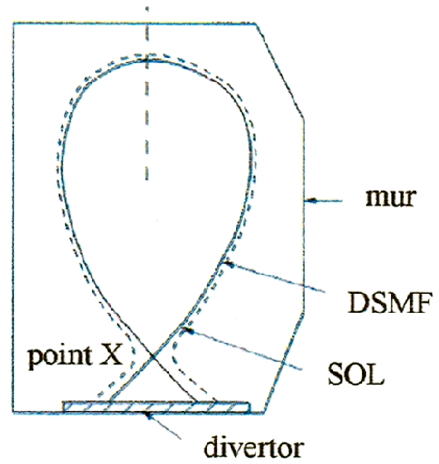
(Last Closed Magnetic Surface)

SOL : Scrape off layer

Limiteur

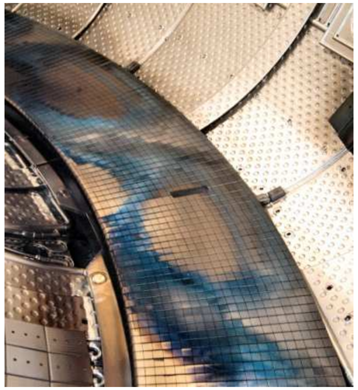


Divertor



Tore **S**upra (**TS**)

Cadarache (France)

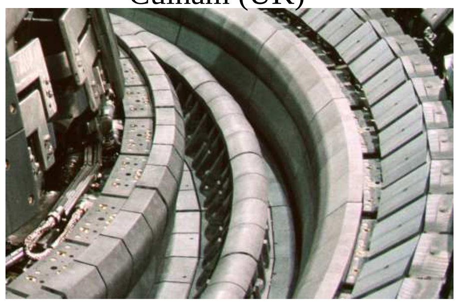


Heat flux of about 10MW/m^2

Heat flux intercepted by the Plasma Facing Component

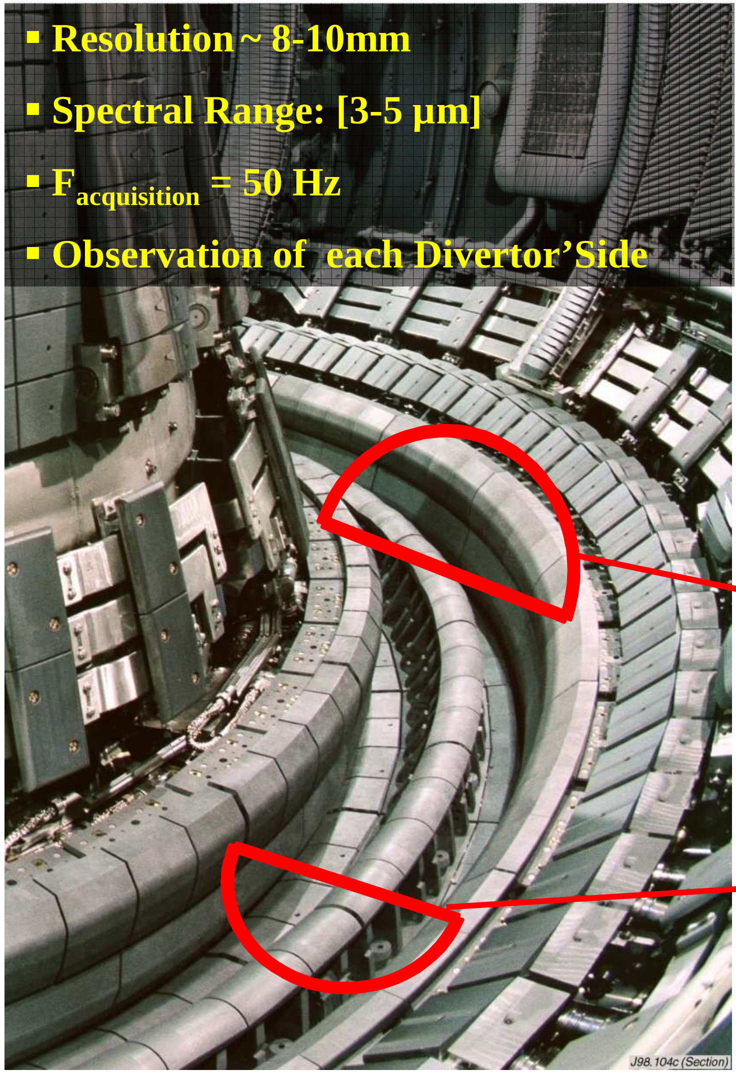
Joint **E**uropean **T**orus (**JET**)

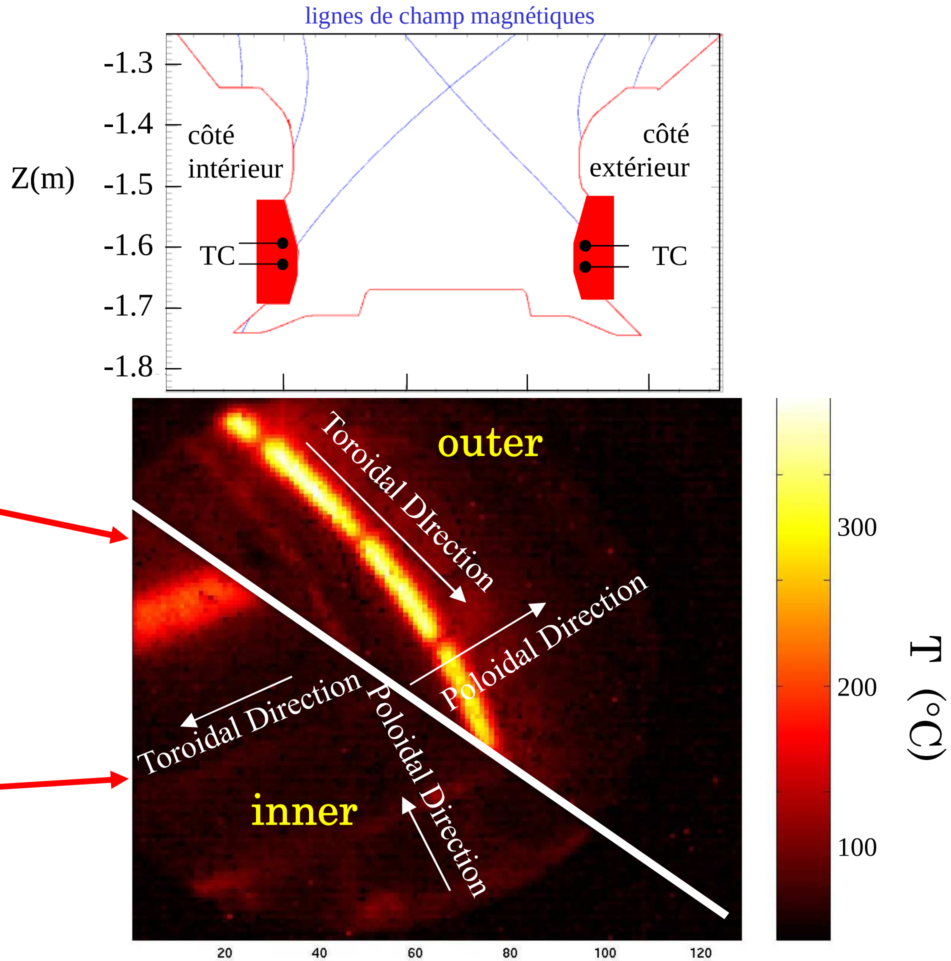
Culham (UK)



JET : Diagnostics and Components

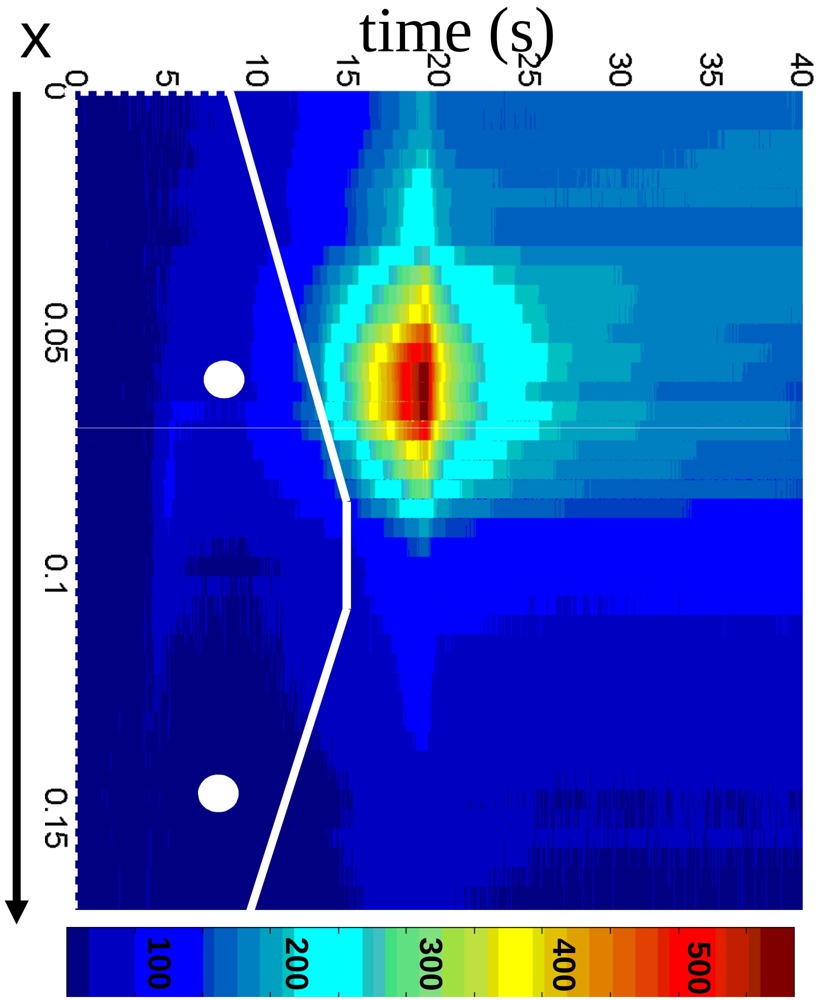
JET Divertor

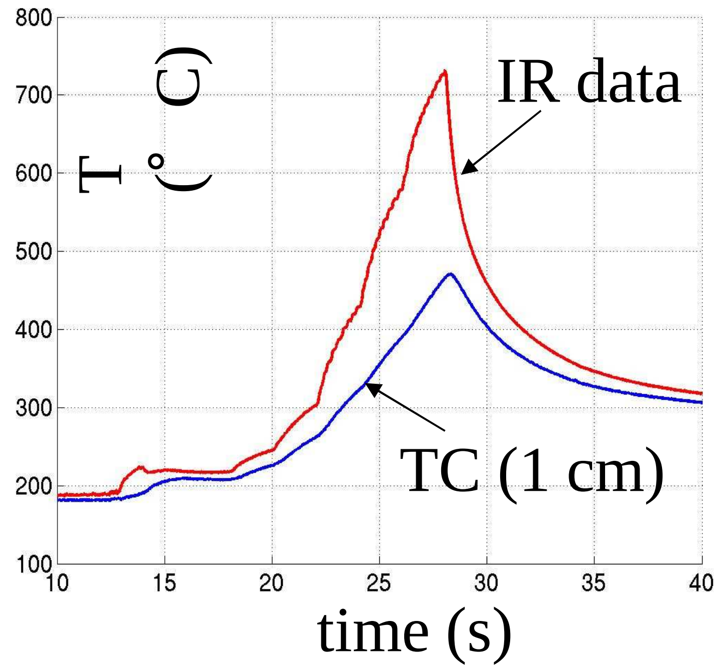




[Gautier, EPS 1997]

Example of Experimental Temperatures





<u>Objectives</u> : Heat flux computation on the plasma facing components

<u>Why ?</u>

- **Critical Heat Flux and temperature of the PFC (10 MW/m², 1200 ° C)**
  - **Components destruction**
  - **Water leak in water-cooled machines**
- **Better understanding of the plasma physic.**

<u>Problem</u>

- **Is the plasma component perfectly known ?**
  - **Dimensions**
  - **Thermal properties**

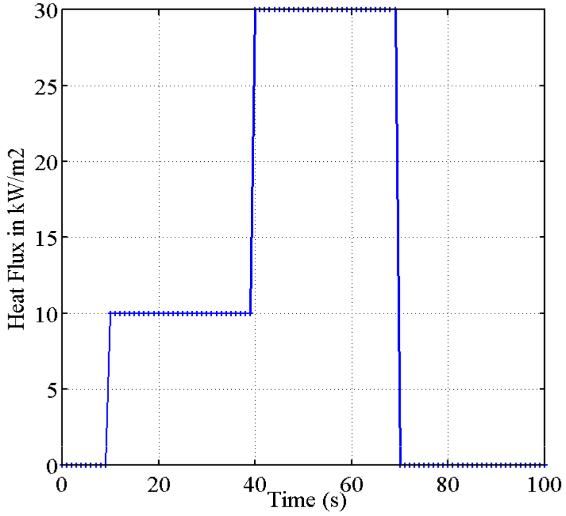
Problems

- **IR data: - Direct computation (surface temperature measurement)**
  - Unknown Thermal Properties

- **TC data: - Spatial resolution**
  - Inverse Problem

We have to use the thermocouple data => Solve an inverse Problem

A simplified version of our problem : 1D, Linear,Semi-Infinite Wall



z

$k = 1\ W/mK$

$Cp = 1000\ J/kg.m^3$

$\rho = 2500\ kg/m^3$

Direct Problem

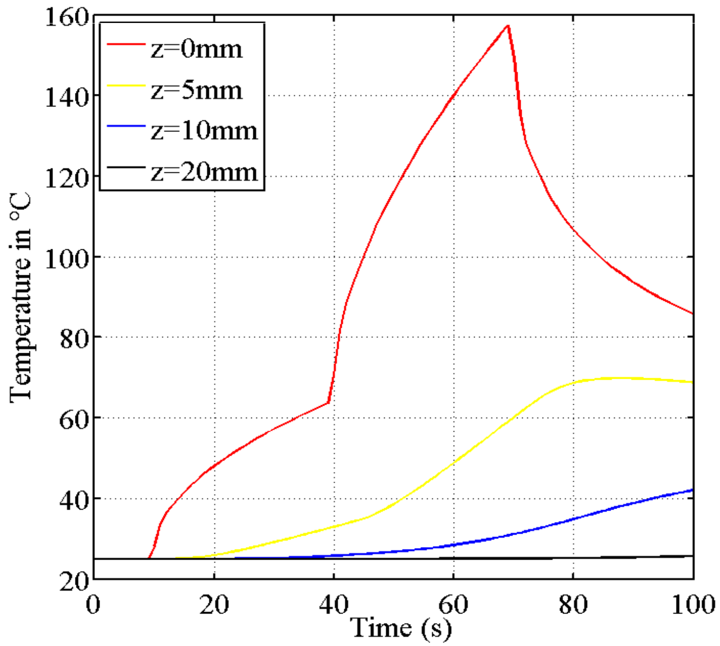
It's possible to have the temperature field *T(z,t)* using:

- **Numerical Simulation**
- **Analytic Solution**
- **Semi-Analytic Solution**

Temperature

+

Noise



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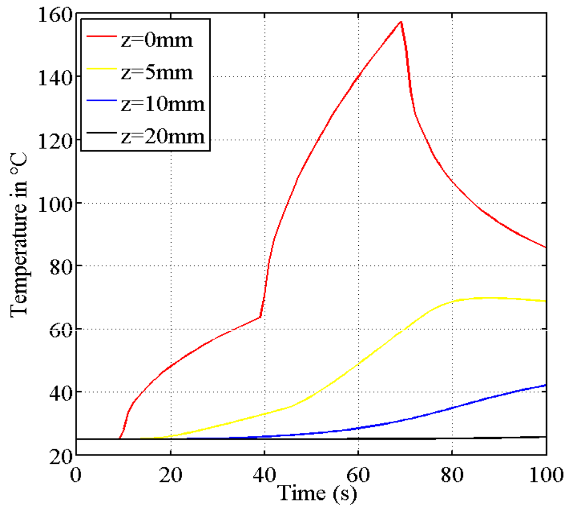
Knowing the thermal properties, is it possible to estimate the heat flux with a temperature measurement ?



+

z

$k = 1\ W/mK$

$Cp = 1000\ J/kg.m^3$

$\rho = 2500\ kg/m^3$

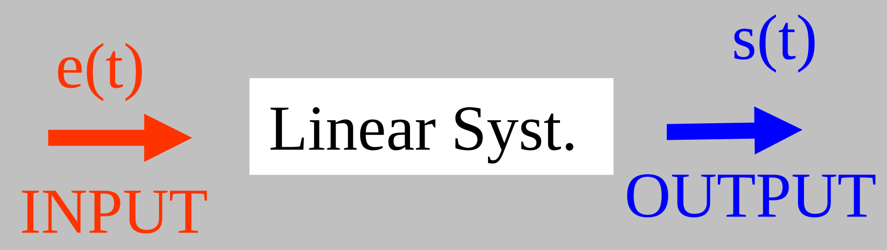
e

Inverse Problem

=> $Q(t)$???

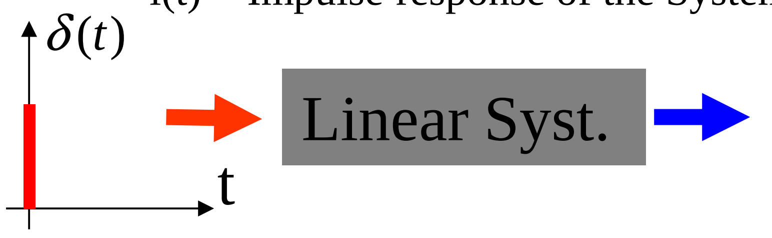
Convolution / Deconvolution

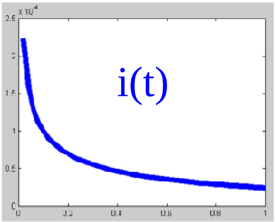
Theory of the Linear System

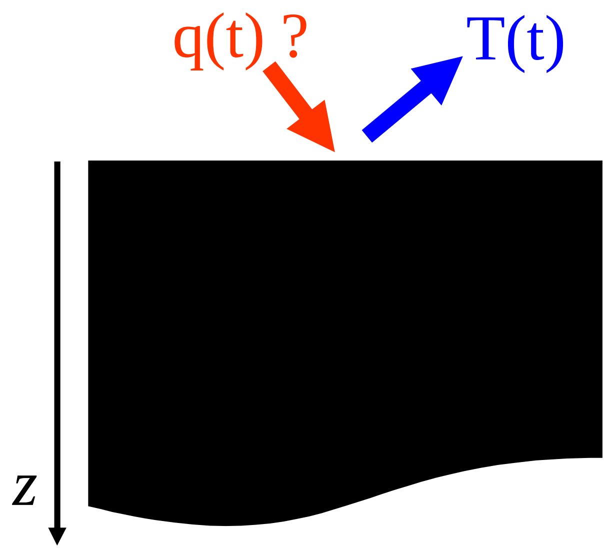


$$s(t) = s(t_0) + e(t) \otimes i(t) = s(t_0) + \int_0^t e(\tau) i(t-\tau) d\tau$$

i(t) = Impulse response of the System





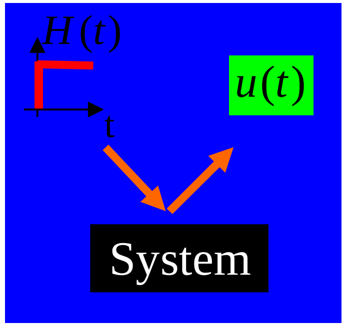


Convolution / Deconvolution

- Duhamel'Theorem:

$$\boxed{T(t)-T_0=Q(t)\otimes i(t)=\int_0^t Q(\tau)i(t-\tau)d\tau}$$

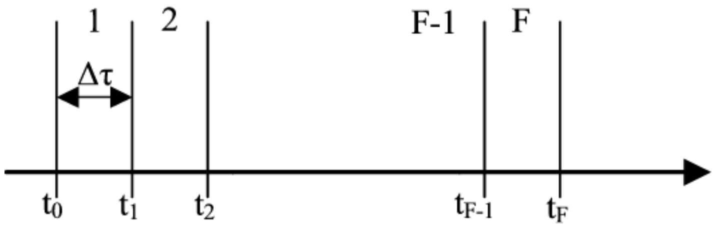
- Step Response:



$$i(t-\tau)=\frac{\partial u(t-\tau)}{\partial t}\approx\frac{u(F-f+1)-u(F-f)}{\Delta\tau}$$

u(t) = Step Response of the System (response to the Heaviside Function)

- Discretization:



F = Number of step time

$$\Delta T_F=T_F-T_0=\sum_{f=1}^{F}(u(F-f+1)-u(F-f))q(f)=\sum_{f=1}^{F}\Delta u_{F-f}q(f)$$

$$\Delta T_F = \sum_{f=1}^{F} \Delta u_{F-f}\, q(f)$$

$$\Delta u_k = u(k+1) - u(k) \qquad 0 \le k \le F-1$$

$$T_1 - T_0 = q_1 \Delta u_0$$
$$T_2 - T_0 = q_1 \Delta u_1 + q_2 \Delta u_0$$
$$T_3 - T_0 = q_1 \Delta u_2 + q_2 \Delta u_1 + q_3 \Delta u_0$$

.

.

$$T_F - T_0 = q_1 \Delta u_{F-1} + q_2 \Delta u_{F-2} + q_3 \Delta u_{F-3} + \ldots\ldots + q_F \Delta u_0$$

F = Number of step time

$$F \quad \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \vdots \\ \vdots \\ \vdots \\ \Delta T_F \end{bmatrix} = \begin{bmatrix} \Delta u_0 & 0 & \cdots & \cdots & \cdots & 0 \\ \Delta u_1 & \Delta u_0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & . & . & 0 \\ \vdots & \vdots & \ddots & \ddots & . & \vdots \\ \vdots & \vdots & . & \ddots & \ddots & \vdots \\ \Delta u_{F-1} & \Delta u_{F-2} & \cdots & \cdots & . & \Delta u_0 \end{bmatrix} . \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ \vdots \\ \vdots \\ q_F \end{bmatrix}$$

Convolution

$$\mathbf{\Delta T} = \mathbf{X} \times \mathbf{Q}$$

Deconvolution

$$\mathbf{Q} = \mathbf{X}^{-1}.\mathbf{\Delta T}$$

The matrix X can be inverted if X is well conditioned

$$K(X) = \|X\| \|X^{-1}\|$$

If K(X) is low => X is well conditioned

If K(X) >> 1 => X is ill conditioned

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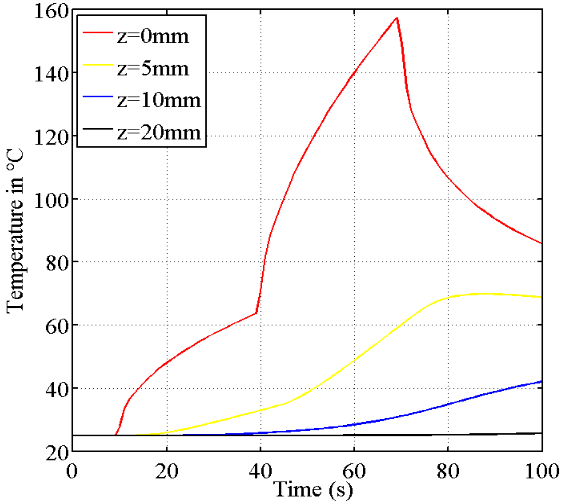

+

z

k = 1 W/mK

Cp = 1000 J/kg.m³

ρ= 2500 kg/m³

e

- Step response computation: analytical solution

$$\Delta T(x,\tau)=\frac{2q_0\sqrt{\tau}}{b\sqrt{\pi}}\left[\exp\left(-\frac{x^2}{4\alpha\tau}\right)-\frac{x}{2}\sqrt{\frac{\pi}{\alpha\tau}}erfc\left(\frac{x}{2\sqrt{\alpha\tau}}\right)\right]$$

- The temperatures are produced with a FEM code

We can solve this problem with Excel or Matlab

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Deconvolution of the temperature at z=5mm: Application with Excel and Matlab

• Why doesn't it work ?

$$\cancel{Q = X^{-1}.\Delta T}$$

=> The matrix is ill conditioned because the problem is ill posed
=> The solution doesn't respect the Stability condition: Q is very sensitive to measurement errors contained in deltaT.

• How to find a solution ?

=> Need to use a regularization procedure
=> Regularization with a penalisation
=> We choose the Tikhonov operator

- Without regularization, the function to minimize is: $\mathbf{J}(\mathbf{X},\mathbf{R},\gamma)=\left\|\mathbf{X}.\tilde{\mathbf{Q}}-\Delta\mathbf{T}\right\|^2$
- With regularization, the new J is:

$$\mathbf{J}(\mathbf{X},\mathbf{R},\gamma)=\left\|\mathbf{X}.\tilde{\mathbf{Q}}-\Delta\mathbf{T}\right\|^2+\gamma\left\|\mathbf{I}.\tilde{\mathbf{Q}}\right\|$$

J is the function to minimise
R is the regularization operator
γ is the regularization parameter

- In our case, we want to limit the norm of $\tilde{\mathbf{Q}}$, so R=I, this is a 0 order regularization.
- To minimise J is equivalent to have the following expression of Q

$$\tilde{\mathbf{Q}}=\left(\mathbf{X}^t\mathbf{X}+\gamma\boldsymbol{I}^t\boldsymbol{I}\right)^{-1}\mathbf{X}^t.\Delta\mathbf{T}$$

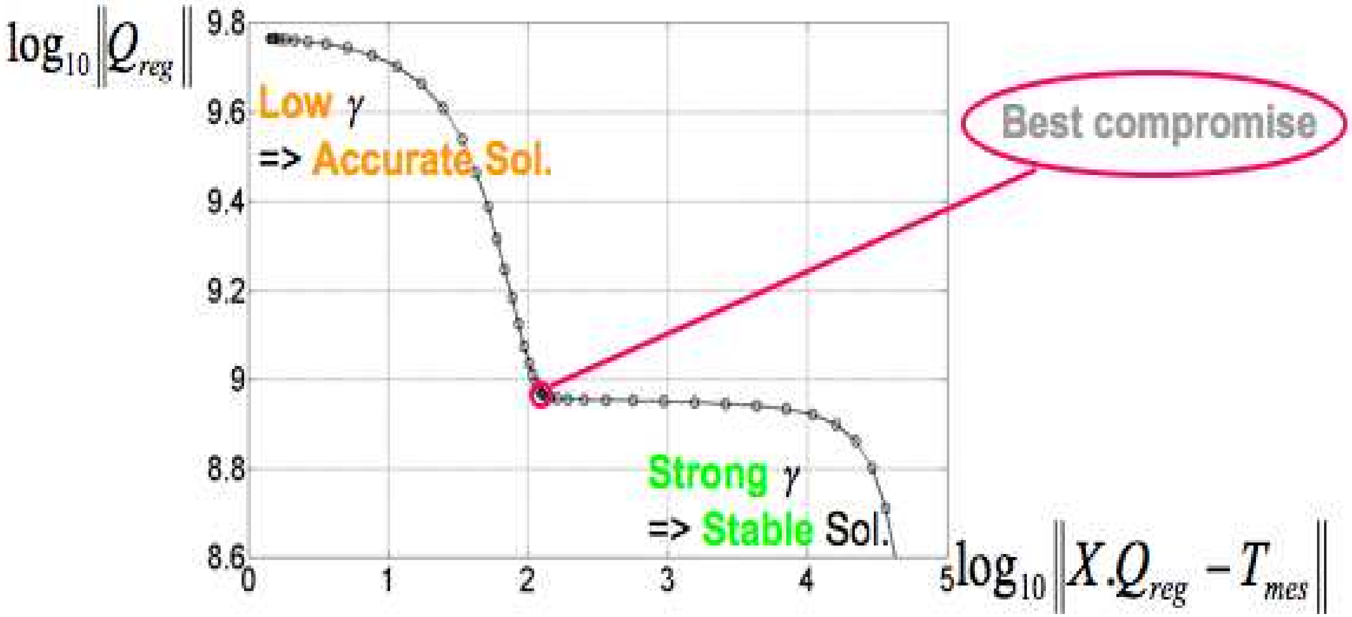
- One can note $\tilde{\mathbf{Q}}$ that is not the exact heat flux but an estimated heat flux; it is a biased value of Q

How to choose γ ?

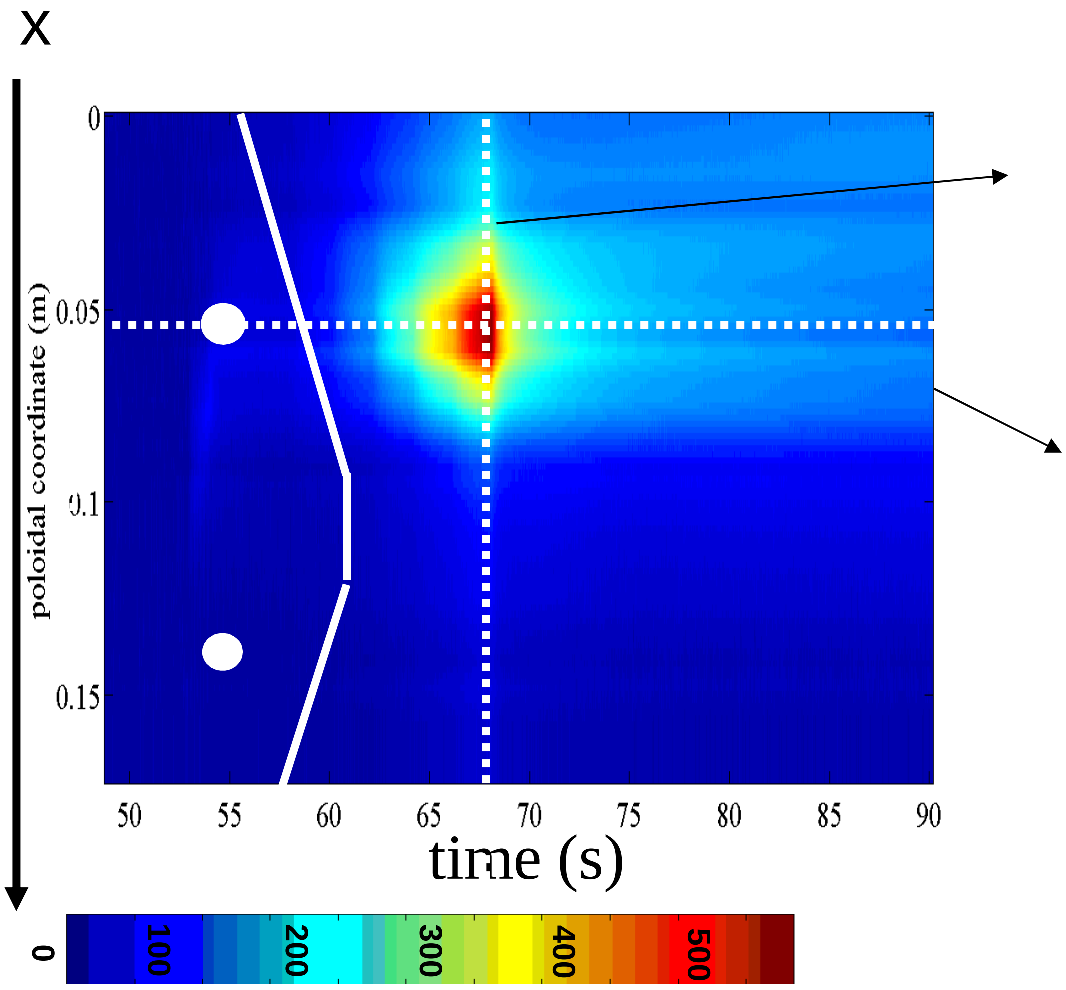
=> γ is chosen to have the best compromise between an exact solution and a stable solution

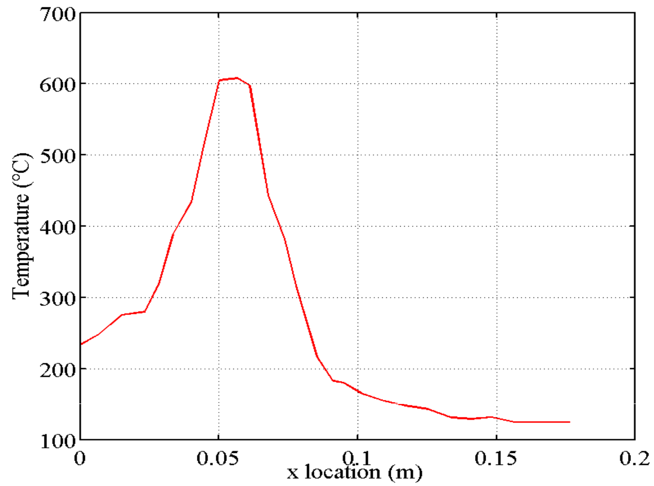
=> Exact solution means that $\left\| \mathbf{X}.\tilde{\mathbf{Q}} - \Delta\mathbf{T} \right\| \to 0$

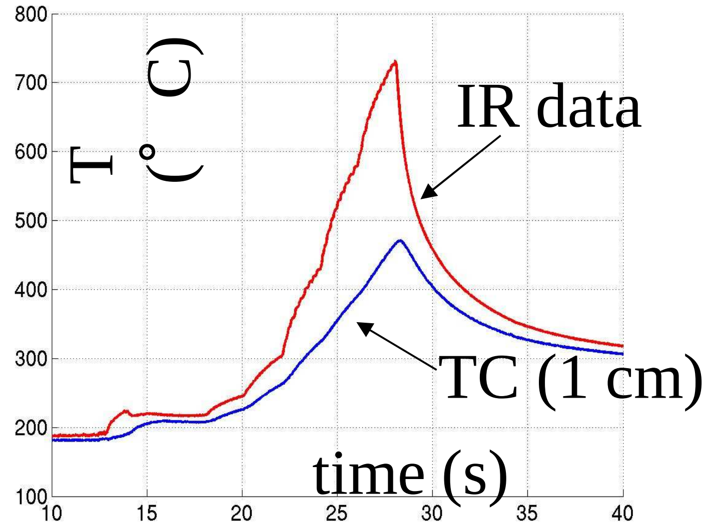
=> Stable means that $\left\| \tilde{\mathbf{Q}} \right\| \to \min$



Application to the tiles of the JET Divertor







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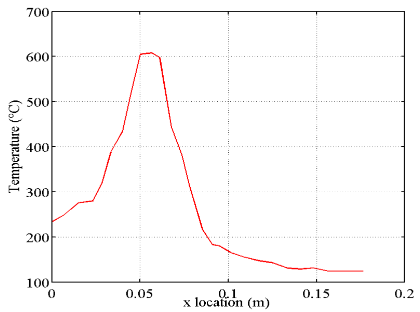
- Assuming that the heat spatial shape is not depending on timed.



$$Q(x,t) = f(x) \, . \, g(t)$$

Deduced from Other Diag.

To compute with the TC data

- Computation of the step response 2D for z=10mm, x=55mm and a spatial shape deduced from the IR data.

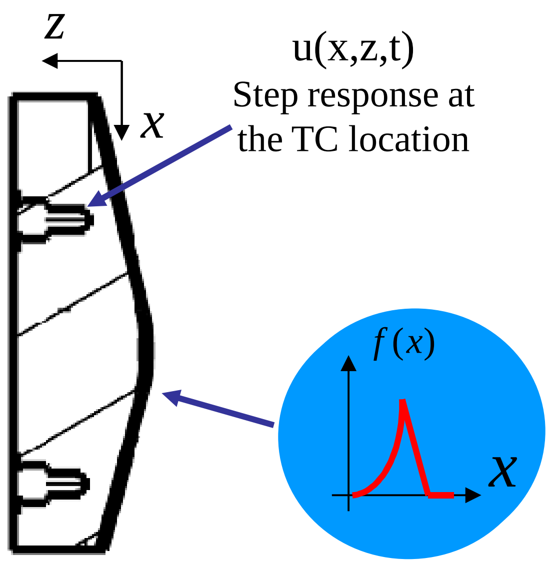
$\Rightarrow$ FEM code

$\Rightarrow$ Semi-analytical method

- Construction of X

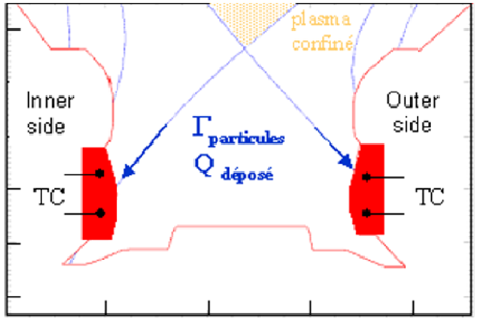
- Construction of $(X^T X + \gamma I)$

- Inversion of $(X^T X + \gamma I)$



Inner Side

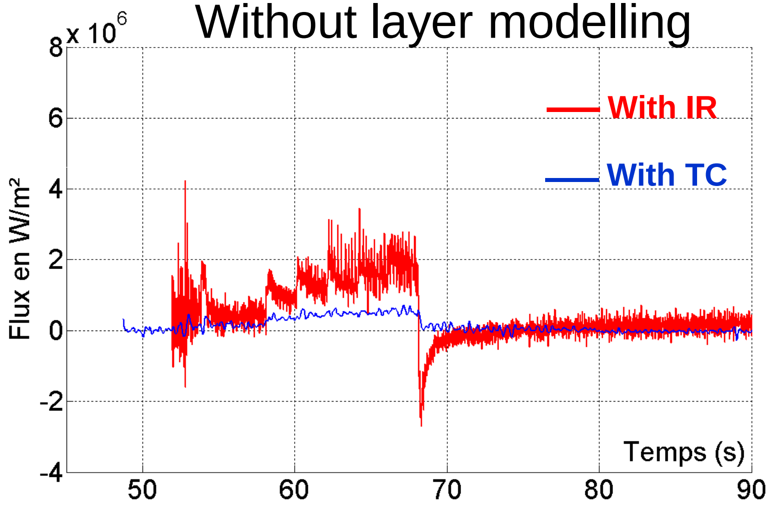
Carbon Layer

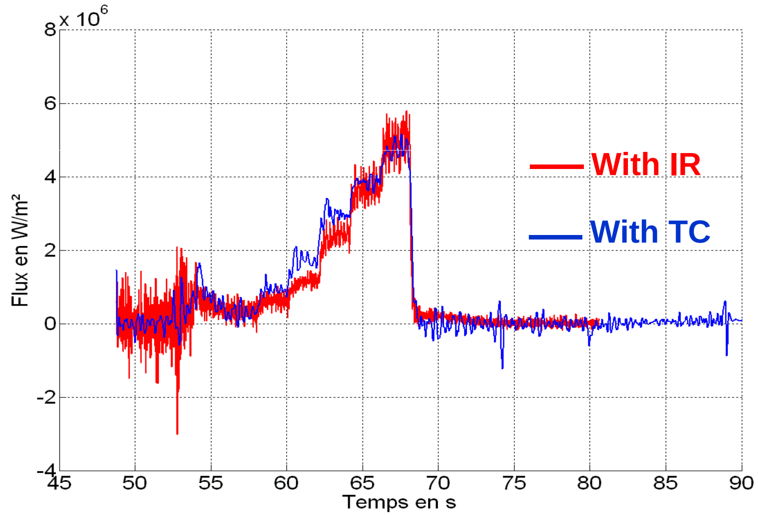


Outer Side

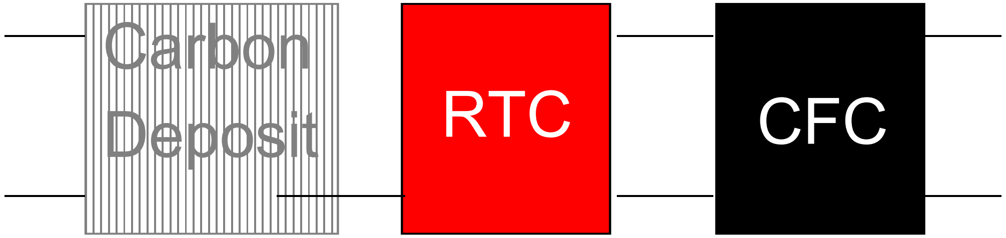
Erosion Zone

Without layer modelling





- Zone with deposit: Flux IR ≠ Flux TC
- Informations on the deposit ?



Sensitive Analysis: $Z_k(t) = \beta_k \dfrac{\partial Q(t,\beta)}{\partial \beta_k}$

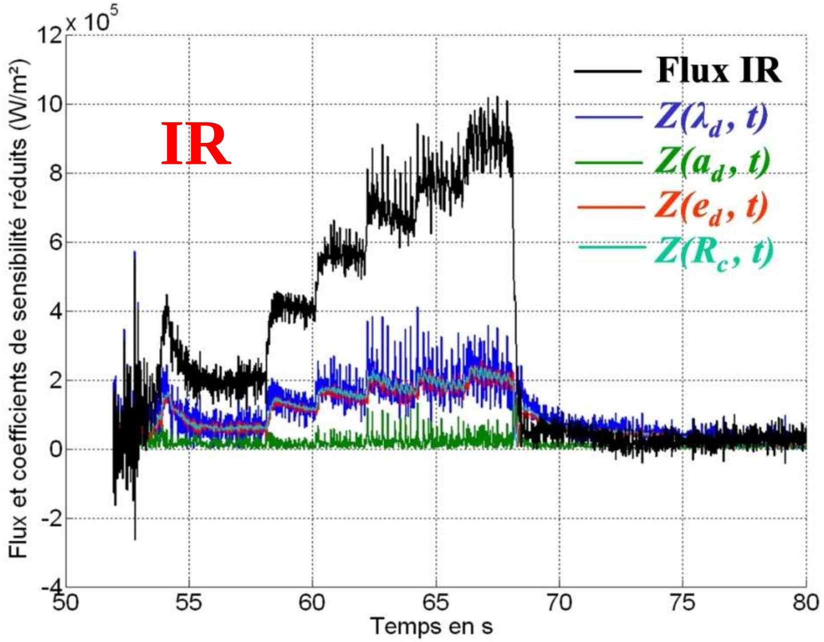
- 4 deposit param. : a_d, λ_d, e_d, Rc
- Very low sensitivity at the TC location
- Sensitive and correlated in surface

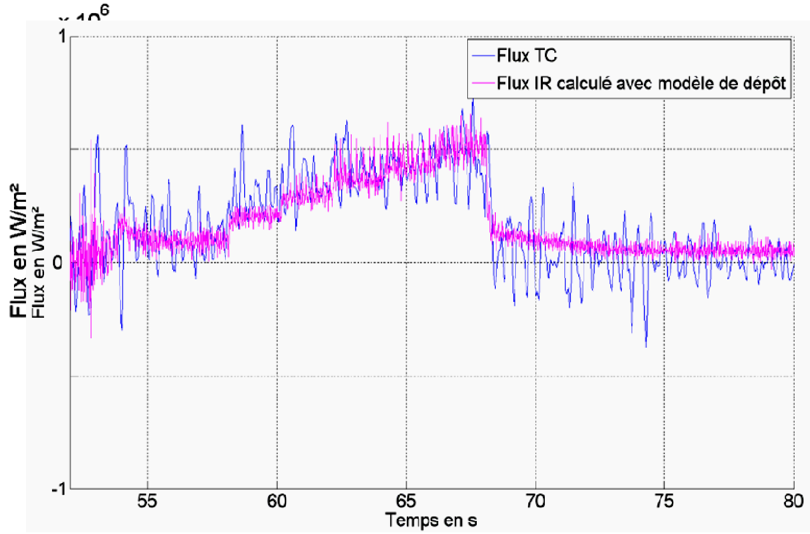
=> Impossible to identify 4 parameters

Identif. Of 1 parameter : R_{eq}

$R_{eq} = 3.10^{-4}$ m²K/W

(h_{eq} = 3.3 kW/m²K)





Conclusion

$$\Delta \mathbf{T}(\boldsymbol{x},\tau)=\frac{2\boldsymbol{q}_0\sqrt{\tau}}{\boldsymbol{b}}\,\boldsymbol{ierfc}\left(\frac{\boldsymbol{x}}{2\sqrt{\alpha\tau}}\right)$$

$$\Delta \mathbf{T}(\boldsymbol{x},\tau)=\frac{2\boldsymbol{q}_0\sqrt{\tau}}{\boldsymbol{b}\sqrt{\pi}}\left[\exp\left(-\frac{\boldsymbol{x}^2}{4\alpha\tau}\right)-\frac{\boldsymbol{x}}{2}\sqrt{\frac{\pi}{\alpha\tau}}\,\boldsymbol{erfc}\left(\frac{\boldsymbol{x}}{2\sqrt{\alpha\tau}}\right)\right]$$